# Mutex Graphs and Multicliques: Reducing Grounding Size for Planning

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We present an approach to representing large sets of *mutual exclusions*, also known as *mutexes or mutex constraints*. These are the types of constraints that specify the exclusion of some properties, events, processes, and so on. They are ubiquitous in many areas of applications. The size of these constraints for a given problem can be overwhelming enough to present a bottleneck for the solving efficiency of the underlying solver. In this paper, we propose a novel graph-theoretic technique based on *multicliques* for a compact representation of mutex constraints and apply it to domain-independent planning in ASP. As computing a minimum multiclique covering from a mutex graph is NP-hard, we propose an efficient approximation algorithm for multiclique covering and show experimentally that it generates substantially smaller grounding size for mutex constraints in ASP than the previously known work in SAT.

### **1** Introduction

Mutual exclusion (mutex) can be traced back to concurrency control, which refers to the condition that prevents simultaneous accesses to a shared resource. In knowledge representation, they specify the constraints that some properties cannot hold at the same time. For example, an object cannot be at different locations at the same time. These constraints frequently occur in applications from model-checking problems in computer-aided verification [2], computer vision [12, 17], graph algorithms [11], and AI planning [14].

The goal of this paper is to develop a graph-theoretic technique for compactly encoding large sets of mutex constraints and apply it to planning in ASP. We do his by focusing on domain-independent AI planning as started out by SATPlan [10]. That is, we will first obtain an ASP planner by a straightforward translation from SATPlan and then study how to encode mutex constraints compactly for the planner.

In SAT/ASP planning, mutex constraints are specified by formulas/rules that, for any state (which involves a time step, also called a *layer* in this paper), the actions with conflicting preconditions or effects, and the fluents that are inferred to be conflicting, are mutually exclusive. A naive encoding of these constraints can certainly generate enough rules to overwhelm the underlying solver for large planning instances. For example, in SAT planning these constraints can be expressed by 2-literal clauses (a 2-literal clause is of the form  $l_1 \vee l_2$  where  $l_1$  and  $l_2$  are literals), which, according to [14], constitute about 50-95% of the formulas, and sometimes they used so much memory that they could not fit in a 32-bit address space.

As shown in [14], significant space-savings can be gained by considering the way in which we encode mutex constraints. We may view the set of mutex constraints on fluents as an undirected graph, called a *mutex graph*, where each fluent is a vertex and each constraint is an edge. When a solver selects one fluent to be true at a given layer, it infers by unit-propagation that each fluent joined directly by an edge

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with the selection must be false. Thus, the set of fluents which are true at a given layer constitutes *an independent set* on the mutex graph.<sup>1</sup>

Rintanen [14] shows that there exist other smaller encodings besides the naive approach of listing out every individual binary constraint and implies that since these encodings are smaller, they must be superior. In their experiments, they use instances of the AIRPORTS domain from an IPC planning competition. This domain is notable because of the vast number of mutex constraints it generates. The larger instances of this problem emit complex mutex graphs which can overwhelm the underlying SAT solver if encoded naively (in a one-constraint-per-edge fashion).

Rintanen further shows that the mutex graphs in these planning problems (even in benchmark AIR-PORTS) tend to be highly structured and that in SAT it is possible to cover the mutex graph (somewhat more compactly) with cliques (complete subgraphs) or with bicliques (complete bipartite subgraphs). A biclique can be expressed in SAT using only one auxiliary variable and one binary clause per assignment. Rintanen demonstrates that cliques can be expressed using only a logarithmic set of bicliques. He concludes that the best way to express a mutex graph in SAT is with a biclique edge-covering.

In this paper, we show that for ASP, cardinality constraints give us more power than is available in SAT and indeed we can directly encode a mutex graph by its clique covering (without the extra cost of a logarithmic factor), but further we can eliminate the choice of whether to use cliques or bicliques entirely and instead cover the graph with *multicliques* (complete multi-partite subgraphs) which is a generalization of both. Indeed, we find that with multicliques, the number of clauses (namely ASP rules) and literals required to encode mutex constraints can be further reduced over Rintanen's results.

The next section provides an ASP planner as the context of dealing with mutex constraints. We also review the definitions of cliques/bicliques and comment on the complexity and representation issues. Section 3 then presents an approximation algorithm for multiclique covering and Section 4 shows how to construct action mutex constraints simultaneously. In Section 5 we present experimental results. Section 6 comments on related work and Section 7 concludes the paper with final remarks.

The ASP encodings in this paper are constructed to run on CLINGO and follow the ASP-Core-2 Standard [4] except that (i) we will use ; to separate rule body atoms since the more conventional comma sign , is overloaded and has a different meaning in more complex rules (CLINGO supports both), and (ii) the disjunctive head of a rule may be written by a conditional literal. The work reported here has been used in a recent construction of a cost-optimal planner in ASP [18].

#### 2 **Preliminaries**

#### 2.1 STRIPS Planning in ASP

We adopt a direct translation of 5 rules of SATPlan [10] into ASP and call the resulting planner ASPPlan.

```
rule 1. holds(F,K) :- goal(F); finalStep(K).
rule 2. happens(A,K-1) : add(A,F),validAct(A,K-1) :- holds(F,K); K > 0.
rule 3. holds(F,K) :- pre(A,F); happens(A,K); validFluent(F,K).
rule 4. :- mutexAct(A,B); happens(A,K); happens(B,K).
rule 5. :- mutex(F,G); holds(F,K); holds(G,K).
```

<sup>&</sup>lt;sup>1</sup>An independent set on a graph is a set of vertices where no two vertices in the set share an edge [16]; equivalently this is a clique in the complement graph.

where validAct(A, K) means that action A can occur at time K and validFluent(F, K) means fluent F can be true at time K.<sup>2</sup> Time steps used in constructing a plan are also called *layers*.

Rule 1 says that goals hold at the final layer. In rule 2, if a fluent holds at layer K, the disjunction of actions that have that fluent as an effect hold at layer K - 1. The next rule says that actions at each layer imply their preconditions. The last two rules are mutex constraints: in rule 4, actions with (directly) conflicting preconditions or effects are mutually exclusive, and in rule 5, the fluents that are inferred to be mutually exclusive are encoded as constraints.

Following SATPlan, we add to our plan "preserving" actions for each fluent. The goal is to simulate the frame axioms by using the existing machinery for having an action add a fluent that gets used some steps later. These preserving actions can be specified as:

```
action(preserve(F)) :- fluent(F).
pre(preserve(F),F) :- fluent(F).
add(preserve(F),F) :- fluent(F).
```

where each fluent *F* has a corresponding *preserving action* denoted by term preserve(F). Preserving actions can be easily distinguished from regular actions. Now that an action occurs at time *K* indicates that its add-effect *F* will hold at time K + 1.

Note that the reason why rule 5 of ASPPlan prevents fluents from being deleted before they're used is a bit subtle. In order for a fluent to hold, it must occur in conjunction with a preserving action at each time step it's held for. A preserving action has that fluent as a precondition and so would be mutex with any action that has it as a delete effect. This means that deleting actions cannot occur as long as that fluent is held (by rule 4).<sup>3</sup>

Like SATPlan, we run this planner by solving at some initial makespan K, where K is the first layer at which *validFluent*(F,K) holds for all *goal*(F), and if it is UNSAT, we increment *finalStep* by 1 until we find a plan.

This is a straightforward and unsurprising encoding in every respect, but has a somewhat surprising consequence as compared to SATPlan. Because ASP models are stable, for any fluent F, holds(F,K) can only be true if there exists some action which requires its truth as per rule 3. Similarly for actions as per rule 2. Furthermore, since rule 2 is disjunctive at every step, the set of actions which occurs is a minimal set required to support the fluents at the subsequent step. This conforms exactly to the approach to planning by Blum and Furst [3]: First build the planning graph, then start from the goal-state planning backwards, at each step selecting a minimal set of actions necessary to add all the preconditions for the current set of actions. That is, in this ASP translation, the neededness-analysis as carried out in [15] is accomplished automatically during grounding or during the search for stable models.

**Smart Encoding of Action Mutexes:** Let us first consider action mutex constraints as expressed by Rule 4 of ASPPlan, which can blow up in size when grounded because nearly any two actions acting on the same fluent can be considered directly conflicting. For example, assume a planning problem in which there is a crane which we must use to load boxes onto freighters and there are many boxes and many freighters available but only one crane. Then we will have one such constraint for every two actions of

<sup>&</sup>lt;sup>2</sup>Blum and Furst [3] give a handy way to identify for each action and each fluent, what is the first layer at which this action/fluent might occur by building the *planning graph*. Note that validAct/2 and validFluent/2 as well as predicates mutexAct/2 and mutex/2 are all extracted from the planning graph.

<sup>&</sup>lt;sup>3</sup>As a further note, when PDDL (planning domain definition language) without any extensions is defined, goals can only be positive and actions can only have positive preconditions. There is a :negative-preconditions extension to PDDL, but we didn't use it. Any problem which uses :negative-preconditions can be trivially adapted to avoid using it by adding a fluent :not-F for every fluent :F and then adding a corresponding add-effect wherever there's a delete-effect and vice versa.

the form, load(Crate, Freighter), for any crate and any freighter. As there is already a quadratic number of actions in the problem description size (*crates*  $\times$  *freighters*), the number of mutex constraints over *pairs* of actions is *quartic* in the initial (non-ground) problem description size.

We would like to avoid such an explosion by introducing new predicates to keep the problem size down. We will only consider two actions to be mutex if one deletes the other's precondition. But we will take extra steps to ensure that no add-effect is later used if the same fluent is also deleted at that step. Here is the revised encoding of rule 4.

```
used_preserved(F,K) := happens(A,K); pre(A,F); not del(A,F).
deleted_unused(F,K) := happens(A,K); del(A,F); not pre(A,F).
:= {used_preserved(F,K); deleted_unused(F,K);
    happens(A,K) : pre(A,F), del(A,F)} > 1; valid_at(F,K).
deleted(F,K) := happens(A,K); del(A,F).
:= holds(F,K); deleted(F,K-1).
```

Effectively, we are splitting the ways in which we care that an action A can relate to a fluent F into three different cases: (i) A has F as a precondition, but not a delete-effect; (ii) A has F as a delete-effect,

but not a precondition; and (iii) A has F as both a precondition and a delete-effect.

By explicitly creating two new predicates for properties (i) and (ii), we have packed this restriction into one big cardinality constraint. Further, we must account for conflicting effects, so we define one more predicate (deleted/2) which encapsulates the union of all actions from properties 2 and 3 (those that delete F) and assert that F cannot hold at this step if any of those actions occurred in the previous one.<sup>4</sup>

#### 2.2 Cliques and Bicliques

We review the definitions of cliques and bicliques and comment on their possible encodings in SAT.

Let G = (V, E) be an undirected graph. A *clique* is a subgraph (C, E') of G such that  $C \subseteq V$  and  $E' = \{(v, u) \in E \mid v, u \in C, u \neq v\}$ . A *biclique* is a subgraph (C, C', E') of G such that  $C, C' \subseteq V, C \cap C' = \emptyset$ , and  $E' = \{(u, v) \in E \mid u \in C, v \in C'\}$ .

That is, cliques are complete subgraphs of a graph and bicliques are complete bipartite subgraphs of a graph. Deciding if a graph has a clique of size n is known to be NP-complete [6, 9]. This is also the case for bicliques under several size measures [6, 13, 19]. There are approximation algorithms for the computation of cliques and bicliques, with approximation guarantees [7], or without [14].

In SAT, given *n* fluents, besides the naive  $O(n^2)$  size representation, cliques can be represented in size O(n) using O(n) many auxiliary variables, or in size  $O(n \log n)$  using only  $O(\log n)$  many auxiliary variables. Bicliques enjoy a more compact representation: if *C* and *C'* form a biclique, then  $|C| \times |C'|$  many binary constraints can be represented by |C| + |C'| many 2-literal clauses using only one auxiliary variable [14]. The idea is that for any literals  $l \in C$  and  $l' \in C'$ , mutex constraints of the form  $l \vee l'$  can all be represented using one new variable, say *x*, by  $\neg l \rightarrow x$  and  $x \rightarrow l'$ .

<sup>&</sup>lt;sup>4</sup>There is a minor difference between the definition of mutex as given in [3], which appears to be overly restrictive, and the definition we're using. Whereas graphplan treats any two actions as mutex if they have conflicting effects (one adds a fluent which the other deletes), we only consider them to be mutex if they have conflicting effects and the add-effect is used at that layer. So we allow actions to occur simultaneously with conflicting effects as long as the relevant fluent doesn't hold afterwards.

# 3 An Approximation Algorithm for Multiclique Covering

In this section, we formulate a polynomial-time, approximation algorithm for multiclique covering. First, let us have a formal definition of multiclique.

**Definition 3.1** Let G = (V, E) be an undirected graph. A multiclique of G is a subgraph  $(C_1, \ldots, C_k, E')$  of G, such that  $C_1 \cup \cdots \cup C_k \subseteq V$ ,  $C_i \cap C_j = \emptyset$  for all  $1 \le i, j \le k$  where  $i \ne j$ , and  $E' = \{(u, v) \in E \mid u \in C_i, v \in C_j, i \ne j\}$ .

*We call each*  $C_i$   $(1 \le i \le k)$  *above a* partition.

**Proposition 3.1** A multiclique is a graph whose complement is a cluster graph, i.e., a set of disjoint cliques.

The claim is easy to verify. Consider any graph G which is a multiclique by definition. In the complement graph  $G^c$ , every partition is a clique. Further, since any two vertices u and v must have an edge if they belong to separate partitions in G, it follows that there are no edges between partitions in  $G^c$ , therefore, the only edges in  $G^c$  belong to cliques. Similarly, if  $G^c$  is a cluster-graph, then the connected components form the partitions in G as a multiclique.

Given a mutex graph, a naive encoding of mutex constraints in ASP is to list each edge between two vertices by a 2-literal constraint. With a multiclique covering, mutex constraints in a mutex graph can be encoded compactly.

Given a graph G = (V, E), the goal of *multiclique covering* is to produce a sequence of multicliques  $\Pi = (S_1, \ldots, S_n)$  for some *n*, where each  $S_i$  is a multiclique subgraph of *G*, for all j > i,  $S_j$  contains at least one edge not in  $S_i$ , and the union of edges in  $S_k$   $(1 \le k \le n)$  is *E*. In the multiclique covering  $\Pi$ ,  $S_i$  and  $S_j$  may share some vertices. In general, to cover all mutex constraints in a mutex graph, the edges covered in different multicliques in  $\Pi$  need not be non-overlapping. In our algorithm, we do allow overlapping if it leads to more compact representation. In summary, as the edges in a mutex graph represent constraints, multiclique covering is to cover the edges of the mutex graph where the edges are spread out in multicliques that are constructed.

For each multiclique constructed, we can encode a constraint graph in ASP as:

```
% Covering is given by inPartition(F,P) if fluent F belongs to partition P,
% and inMulticlique(P,M) if partition P belongs to multiclique M.
% p(P,K): P is a partition at layer K.
partitionHolds(P,K) :- holds(F,K); inPartition(F,P).
:- {p(P,K): partitionHolds(P,K),inMulticlique(P,M)} > 1;
multiclique(M); layer(K).
```

Here we have a cardinality constraint expressing the rule that among all partitions P of multiclique M, at most one holds at layer K. Furthermore, if any fluent F holds at layer K, so does its partition P.

Additionally, we can avoid some unnecessary rules by handling singleton partitions specially. A singleton partition can be packed *directly* into the cardinality constraint rather than introduced through an auxiliary atom:

```
:- {partitionHolds(P,K):inMulticlique(P,M);
            holds(F,K):singletonPartitionOf(F,M)} > 1;
            multiclique(M); layer(K).
```

Now, our ASPPlan given in Section 2.1 is updated by replacing Rule 5 therein with the above rules.

Algorithm I Multiclique Covering							
1:	<b>procedure</b> FIND_COVER( $g$ :: Graph) $\rightarrow$ Set MultiClique						
2:	var $uncovered \leftarrow g.edges ::$ Set Edge						
3:	var <i>multicliques</i> $\leftarrow$ {} :: Set MultiClique						
4:	while uncovered.nonempty do						
5:	<i>new_multiclique</i> $\leftarrow$ NEXT_MULTICLIQUE()						
6:	$multicliques \leftarrow multicliques \cup \{new\_multiclique\}$						
7:	$uncovered \leftarrow uncovered \setminus EDGES\_COVERED\_BY(new\_multiclique)$						
8:	end while						
9:	return multicliques						
10: end procedure							

Hence, given a planning instance, if we can construct a multiclique covering from its mutex graph, we can use ASP to encode these constraints compactly. Now let us find an algorithm for this task.

In general, finding a minimum multiclique covering (using as few multicliques as possible) is NPhard. To see this, consider the problem of finding a minimum multiclique covering on a bipartite graph. It's easy to see that a multiclique on a bipartite graph is a biclique. Thus the minimum multiclique covering of a bipartite graph is the minimum biclique covering. The size of the minimum biclique covering of a bipartite graph is also known as its *bipartite dimension*. Finding the bipartite dimension of a graph is known to be NP-hard [1]. Thus, finding a minimum multiclique cover is also NP-hard.

Nonetheless, we can still use approximation algorithms similar to those used in [7]. One critical observation is that under the restriction that a multiclique must use exactly a particular set of vertices, there is always only one optimal way to partition those vertices into a multiclique to cover a maximal set of edges: If there is a path between two vertices v and w in the complement of the induced graph, then they must belong to the same partition. If there is no path, then we might as well put them in separate partitions. Therefore, the best partition is the one which makes a partition for each connected component in the complement of the induced graph.

Let us use an example to illustrate. Consider the mutex graph G = (V, E) on the left of the figure below and its complement graph  $G^c$  on the right. The connected components of  $G^c$  give us a multiclique  $\{\{a,b,d\},\{c\},\{e\}\}\)$ , which covers almost all edges in E except edge (a,b). So edge  $(a,b) \in E$  will have to be captured in another multiclique.



Figure 1: An example mutex graph and its complement.

Our algorithm is given in Algorithm 1, with supporting functions given in Algorithm 2. The algorithm is greedy, simple, and polynomial-time. We track the set of uncovered edges and tack multicliques on one at a time, greedily building each multiclique in such a way so as to maximize the difference  $\phi_1 - \phi_2$ , where  $\phi_1$  is the number of literals in the naive encoding and  $\phi_2$  is the number of literals in our ASP encoding of the corresponding multiclique. A difference indicates an encoding reduction.

Algorithm 2 Multiclique Covering Helper Functions 1: type MCPartition = Set Vertex 2: type MultiClique = Set MCPartition 3: **function** MAKE\_MULTICLIQUE(vs :: Set Vertex)  $\rightarrow$  MultiClique **return** g.induced\_subgraph(vs).complement().connected\_components() 4: 5: end function 6: **function** EDGES\_COVERED\_BY(mc :: MultiClique)  $\rightarrow$  Edge **return**  $\{(x, y) | p \in mc, q \in mc, p \neq q, x \in p, y \in q\}$ 7: 8: end function 9: function COUNT\_UNCOVERED\_INCIDENT\_EDGES(x :: Vertex)  $\rightarrow \mathbb{N}$  $|(g.incident\_edges(x) \cap uncovered)|$ 10: 11: end function 12: **procedure** DEFAULTS\_FOR(*vs* :: Set Vertex)  $\rightarrow$  MCPartition 13: candidates  $\leftarrow \bigcap \{g.neighbors(v) \mid v \in vs\}$  :: Set Vertex **return** { $c \mid c \in candidates, |g.incident\_edges(c) \cap uncovered| \ge 2$ } 14: 15: end procedure 16: **procedure** SCORE(*vs* :: Set Vertex)  $\rightarrow \mathbb{Z}$ *multiclique* :: MultiClique 17:  $multiclique \leftarrow MAKE\_MULTICLIQUE(vs) \cup DEFAULTS\_FOR(vs)$ 18: 19: *newly\_covered* :: Set Edge  $newly\_covered \leftarrow EDGES\_COVERED\_BY(multiclique) \cap uncovered$ 20:  $complexity\_cost :: \mathbb{Z}$ 21:  $complexity\_cost \leftarrow \sum_{pmulticlique} \begin{cases} 1 & \text{if } |p| = 1\\ 2*|p|+1 & \text{if } |p| > 1 \end{cases}$ 22: 23: **return** 2 \* |*newly\_covered*| - *complexity\_cos* 24: end procedure 25: **procedure** NEXT\_MULTICLIQUE  $\rightarrow$  MultiClique 26: *first\_vertex* :: Vertex *first\_vertex*  $\leftarrow$  arg max<sub>*g,vertices*</sub>( $\lambda w$ . COUNT\_UNCOVERED\_INCIDENT\_EDGES(w)) 27: var *vertex\_set*  $\leftarrow$  {*first\_vertex*} :: Set Vertex 28: repeat 29: *next* :: Vertex 30: *next*  $\leftarrow$  arg max<sub>g.vertices</sub>( $\lambda w$ . SCORE(*vertex\_set*  $\cup$  {w}))) 31: *improved*  $\leftarrow$  SCORE(*vertex\_set*  $\cup$  {*next*}) > SCORE(*vertex\_set*) 32: if improved then 33:  $vertex\_set \leftarrow vertex\_set \cup \{next\}$ 34: end if 35: until improved 36: **return** MAKE\_MULTICLIQUE(*vertex\_set*  $\cup$  DEFAULTS\_FOR(*vertex\_set*)) 37: 38: end procedure

For more details, in Algorithm 1, the variable *multicliques* is empty to start with. Then it iteratively adds one new multiclique at a time until all edges are covered.

In the helper function NEXT\_MULTICLIQUE in Algorithm 2, we select the first vertex by finding the one incident to the most uncovered edges. This is accomplished at Line 27 (line numbers below all refer to Algorithm 2), where we use a lambda function which is applied to each vertex for the parameter *w*. We then repeatedly select each subsequent vertex to greedily maximize the size difference mentioned above under the assumption that we will finish by adding on a "default partition" of vertices, until no improvement can be generated (lines 29-36 of Algorithm 2). The default partition consists of all vertices which have an edge to every vertex we have selected so far including at least two edges not yet covered (lines 12-15).<sup>5</sup>

Given a set of vertices vs, the function MAKE\_MULTICLIQUE(vs) generates a multiclique, where the partitions are obtained by finding the connected components of the complement graph induced from vs, along with the covered edges (lines 3-5).

Note that, instead of *removing* edges from the graph once they've been assigned to a multiclique, we keep a separate record of "uncovered" edges which still remain to be assigned. In this way the same edge may be covered twice by different multicliques if that helps minimize the encoding (cf. line 27).

**Theorem 3.1** Given a mutex graph G = (V, E), the algorithm FIND\_COVER terminates after a number of execution steps in polynomial time in the size of G, and after termination, a sequence of multicliques  $\{(V_1, E_1), \dots, (V_n, E_n)\}$  is generated such that  $V_1 \cup \dots \cup V_n \subseteq V$  and  $E_1 \cup \dots \cup E_n = E$ .

**Proof:** First, we verify that each  $(V_i, E_i)$   $(1 \le i \le n)$  is a multiclique.  $V_i$  is returned as a set of vertices by NEXT\_MULTICLIQUE and partitioned by MAKE\_MULTICLIQUE into  $C_1, \ldots, C_k$  satisfying the following statement: for any  $i \ne j$ ,  $v \in C_i$  and  $v' \in C_j$  iff there is no path between v and v' in the complement of the graph induced from  $V_i$  iff there is an edge between v and v' in the given mutex graph. Hence, each vertex in any partition is connected to every vertex in a different partition. Then, to obtain a multiclique, we only need to let  $E_i$  be the set of edges that connect vertices of different partitions.

The algorithm terminates since each  $E_j$  covers at least one of the uncovered edges. The first vertex is selected such that it maximizes the number of uncovered edges to which it's incident, so as long as there are uncovered edges, we're guaranteed to select a first vertex which is incident to at least one of them. Trivially we can extend this to a multiclique which covers an uncovered edge by selecting the vertex on the other side of any one of them for a score of at least zero. Since the score of the multiclique is only allowed to improve from there and the score measures the number of uncovered edges we've covered, it must be the case that every multiclique will cover at least one new uncovered edge (otherwise its score would be negative).

The claim on polynomial time holds because the number of multicliques is bounded by |E| and there are at most |E| calls to NEXT\_MULTICLIQUE; further, it can be easily checked that the computation of each such call takes polynomial time.

Let's take a look at how this behaves on an example graph. We'll start with a mutex graph for a ferry crossing problem in which we have three islands, a ferry and a car. The ferry can be at any of the three islands and it can have just moved or be in the process of loading. The car can be on the ferry or at one of the three islands. If loading then the car is not currently on the ferry. Figure 2 shows what the mutex graph for the problem looks like.

<sup>&</sup>lt;sup>5</sup>If there is only one, there will be no savings in encoding size, as it would require the same number of literals/rules to include a vertex in a partition.



Figure 2: Mutex graph for the ferry problem.

Now let's run our multiclique cover algorithm on it. We get:

```
% Multiclique 0 has all singleton parts
:- {holds(just_moved(ferry,island_a),T);
    holds(just_moved(ferry,island_b),T);
    holds(just_moved(ferry, island_c),T);
    holds(loading(ferry),T)
    \} > 1; step(T).
% Multiclique 1 has all singleton parts
:- {holds(car_at(island_a),T);
    holds(car_at(island_b),T);
    holds(car_at(island_c),T);
    holds(on_ferry(car),T)
    } > 1; step(T).
% Multiclique 2 has three non-singleton partitions
partitionHolds(part(2,0),T) :- holds(ferry_at(island_a),T).
partitionHolds(part(2,0),T) := holds(just_moved(ferry,island_a),T).
partitionHolds(part(2,1),T) :- holds(ferry_at(island_b),T).
partitionHolds(part(2,1),T) :- holds(just_moved(ferry,island_b),T).
partitionHolds(part(2,2),T) :- holds(ferry_at(island_c),T).
partitionHolds(part(2,2),T) :- holds(just_moved(ferry,island_c),T).
```

```
:- {partitionHolds(part(2,0),T);
    partitionHolds(part(2,1),T);
    partitionHolds(part(2,2),T)} > 1; step(T).
% Multiclique 3 has two singleton parts and so is just a normal
% mutex constraint.
```

```
:- holds(loading(ferry),T); holds(on_ferry(car),T).
```

In total we have (per-layer) a grounded 10 rules with 25 literals. Had we used the naive encoding it would have been 22 rules with 44 literals so we can see this encoding is quite a bit more compact.

To give a better picture, in Figure 3 we color each edge with the multiclique to which it belongs. Note that three of the edges ended up in two distinct multicliques and so are duplicated in the image:



Figure 3: Colored multiclique covering for the ferry problem.

# **4** Eventual Fluent Mutex Constraints

In Section 2.1 we found a way for the ASP solver to avoid explicitly dealing with action mutex constraints and so were able to save on grounded encoding space. But we still have a problem because the algorithm presented by Blum and Furst [3] for *generating* fluent mutex constraints in the first place requires simultaneously constructing action mutex constraints.

Indeed, Rintanen [14] reports being unable to run experiments on the largest AIRPORTS instances from IPC-2004 because the action mutex constraints used so much memory they wouldn't fit in a 32-bit address space.

In this section, we find a way to circumvent this problem and were able to generate mutex constraints on the very largest (AIRPORTS-50) instance while using only about a gigabyte of memory.

Mutex constraints as defined in [3] are "per-layer". You determine the set of mutex constraints *at each layer* by looking at what actions, fluents and mutex constraints were in the previous layer. Two actions are mutex if they are directly mutex or have any mutex preconditions. Two fluents are mutex if all respective pairs of causing actions are mutex. However, suppose we only care to discover and encode which fluents are *always* mutex in the sense that for *every* layer up to an arbitrarily large makespan they cannot both be true.

One way to obtain this set is to build the planning graph outward until the set of mutex constraints stabilizes. That is, we can stop once we find two consecutive layers at which the set of mutex constraints doesn't change. But this would still require tracking action mutex constraints for all pairs of actions.

The key insight is that fluents which are always mutex will be so in *sequential* planning (where exactly one action happens at each layer) as well as in parallel planning. A parallel plan is just a way of compressing a sequential plan into fewer steps so the set of pairs of things which can be true at some point will be the same regardless of how we express it.

Since a sequential plan can be expressed as a parallel plan where at most one non-preserving action happens at each layer, we can run the mutex generation algorithm under the assumption that *all* non-preserving actions are mutex with each other. Then we only need to explicitly keep track of which actions are mutex with each of the *preserving* actions. There are generally significantly fewer preserving actions than total actions. When the set of mutex fluent-pairs stabilizes, it should come out the same as if we had obtained these pairs by building the planning graph normally and waiting for the mutex fluents to stabilize.

#### **5** Experiments

We implemented the multiclique generation algorithm in Haskell, representing a fluent or action as an *Int* and a collection of mutex constraints as an *IntMapIntSet*. Both *IntMap* and *IntSet* come from the *containers* package. A partition of a multiclique was represented as an *IntSet*, a multiclique as a list of partitions, and a multiclique covering as a list of multicliques.

We ran this algorithm on the same instances as Rintanen (as well as on the AIRPORTS-50 instance, the largest problem in the set) and found a significant improvement over his results. Note that these edge-counts do *not* take into account neededness. That is, they cover many fluents and actions which are irrelevant to the goal of the problem and are guaranteed not to be explored by the solver. When we accounted for neededness we found the graphs got much smaller (approximately 5-fold). But we chose not to utilize this so that our results would be better comparable to Rintanen's.

In Table 5, "Edges" is the number of edges in the mutex graph for each instance. "CL" is the number of grounded clauses (rules) we used to encode this graph. These clauses are a mix of binary constraints and "at most 1" cardinality constraints. Because not all the clauses are binary, we are compelled to give the sum number of literals among all the constraints. This is the "Lit" column.

During our experiments, after a look at a couple of example instances, it became immediately clear to us that the majority of edges belong to the first few multicliques found. After that the number of edges covered per clause drops off rapidly. Thus, if we are willing to forget a small percentage of the edges, we can reduce the number of clauses necessary to encode the graph much further. For each instance, we reran the multiclique generation algorithm terminating it as soon as it had covered 90% of the total number of

Instance	Edges	CL	Lit	Edges*	CL*	Lit*	R-Lit	
AP-21	181884	7531	16437	166229	2336	4783	26382	
AP-22	275515	11310	25014	249173	3464	7104	42776	
AP-23	371062	14969	33100	336209	4806	9929	63552	
AP-24	373188	15353	33894	337385	4907	10103	60814	
AP-25	467653	18834	41821	421181	6208	12816	83438	
AP-26	566948	22507	50252	511401	8025	16625	100494	
AP-27	571298	22777	50801	514978	8155	16890	107442	
AP-28	669336	26488	59201	602737	9941	20616	132120	
AP-36	324835	9870	21502	297160	3084	6306	37744	
AP-37	490408	14826	32921	442256	4266	8696	61362	
AP-38	487033	14678	32793	438457	4263	8682	58928	
AP-39	654787	20501	45166	598421	6352	12965	89294	
AP-40	656469	20486	45150	599396	6351	12956	87744	
AP-41	653096	20241	44709	588884	5846	11914	84628	
AP-50	2613736	76180	171944	2353222	34538	71644	-	

Table 1: Multiclique Reduction for AIRPORTS (Abbreviated AP)

edges.<sup>6</sup> The resulting numbers of edges covered, clauses, and literals required are given respectively by the columns "Edges\*", "CL\*", and "Lit\*". "R-Lit" gives the number of literals required for Rintanen's biclique encoding. It's twice the number of constraints he reports [14] since all his constraints are binary clauses (having exactly two literals).

It is worth mentioning that our implementation of multiclique covering has been employed in a costoptimal planner in ASP [18]. That is, all the experiment results reported in [18] for that planner used this implementation for the representation of mutex constraints, where every plan produced by the planner was validated by the Strathclyde Planning Group plan verifier VAL [8].

# 6 Related Work

In [14], an algorithm called IDENTIFY-BICLIQUE is presented. Given a graph G = (V, E), the algorithm starts with the trivial biclique  $\emptyset$ , V, and repeatedly adds nodes to the first part. Nodes from the second part are removed if there is no edge between them and the new node in the first part. The nodes are chosen to maximize the size reduction. The algorithm terminates when the size reduction is no longer possible.

Our algorithm on multiclique covering is a natural extension of the IDENTIFY-BICLIQUE algorithm with some key differences.

- We're generating multicliques rather than bicliques so there can be more than two partitions. In contrast with Rintanen's explicit construction of two partitions, which is possible and convenient because of the limit on two, we generate partitions for a multiclique based on a graph-theoretic property.
- As commented earlier, instead of *removing* edges from the graph once they've been assigned to

<sup>&</sup>lt;sup>6</sup>In our coding for the experiments, similar to edges, the uncovered edges are just represented by another *IntMapIntSet*.

a multiclique, we keep a separate record of "uncovered" edges which still remain to be assigned. The multiple uses of the same covered edges can sometimes further minimize the encoding in ASP.

• Instead of optimizing for |Clauses in naive encoding| – |Clauses in our encoding|, we're optimizing for |*Literals* in naive encoding| – |*Literals* in our encoding|.<sup>7</sup>

Although in this paper we have focused on reducing grounding size for ASP planning, our algorithm can be applied to other applications in other knowledge representation languages. We note that in some logic-based knowledge representation languages, such as SAT, encoding multicliques may not be as convenient and compact as can be done in ASP though. This gives ASP a major advantage.

The grounding bottleneck for constraints has been tackled in the literature with different approaches, e.g., by Cuteri et al. [5], where mutex constraints can be added on demand. A comparison with our approach merits a further investigation.

### 7 Conclusion and Final Remarks

Mutex constraints can significantly contribute to the overall grounding size of a planning problem. These constraints can be represented by a mutex graph where vertices are fluents and edges represent exclusiveness. In this paper, we address this problem by proposing an algorithm for a multiclique covering from a given mutex graph. As computing a minimum multiclique covering from a mutex graph is NP-hard, we propose an intuitive, approximation algorithm and show experimentally that it generates substantially smaller grounding size for mutex constraints in ASP than the previously known work in SAT.

Like [14], our approximation algorithm does not provide any approximation guarantees. A question of interest is whether such a guarantee can be formulated and proved.

The benchmark used in the experiments reported in this paper is limited to the AIRPORT problem. Experiments using more benchmarks are needed. Following Rintanen [14], our goal is to seek smaller grounding sizes, under the assumption that smaller grounding sizes are better. This assumption may not hold true in general in all cases, and needs to be tested out by more experiments.

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<sup>&</sup>lt;sup>7</sup>When dealing with strictly binary clauses (as in Rintanen's case), these behave identically since latter metric is just the former multiplied by two.

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