Refining the Semantics of Epistemic Specifications

Ezgi Iraz Su*
Sinop University, Department of Computer Engineering, Sinop, Turkey
eirazsu@sinop.edu.tr

Answer set programming (ASP) is a problem-solving approach, which has been strongly supported both scientifically and technologically by several solvers, ongoing active research, and implementations in many different fields. However, although researchers acknowledged long ago the necessity of epistemic operators in the language of ASP for better introspective reasoning, this research venue did not attract much attention until recently. Moreover, the existing epistemic extensions of ASP in the literature are not widely approved either, due to the fact that some propose unintended results even for some simple acyclic epistemic programs, new unexpected results may possibly be found, and more importantly, researchers have different reasonings for some critical programs. To that end, Cabalar et al. have recently identified some structural properties of epistemic programs to formally support a possible semantics proposal of such programs and standardise their results. Nonetheless, the soundness of these properties is still under debate, and they are not widely accepted either by the ASP community. Thus, it seems that there is still time to really understand the paradigm, have a mature formalism, and determine the principles providing formal justification of their understandable models. In this paper, we mainly focus on the existing semantics approaches, the criteria that a satisfactory semantics is supposed to satisfy, and the ways to improve them. We also extend some well-known propositions of here-and-there logic (HT) into epistemic HT so as to reveal the real behaviour of programs. Finally, we propose a slightly novel semantics for epistemic ASP, which can be considered as a reflexive extension of Cabalar et al.'s recent formalism called autoepistemic ASP.

1 Introduction

Answer set programming (ASP) has been proposed by Gelfond and Lifschitz (GL) [11] as an approach to declarative programming. Its reduct-based GL-semantics is given by *answer sets* (alias, *stable models*)—consistent sets A of ground literals[1] (referred to as *valuations*) in which $p \notin A$ or $\sim p \notin A$ for every atom p, roughly described as the smallest per subset relation, and supported classical models of a program. ASP provides a successful, and relatively simple way of solving problems: first, a problem is encoded as a logic program whose answer sets correspond to solutions. Then, by means of efficient ASP-solvers computing these models, we obtain solutions in the form of answer sets. As a result, currently, ASP has a wide range of applications in science and technology. However, as first recognised by Gelfond [8], ASP is not strong enough to correctly reason about the global situation in the presence of multiple answer sets of a program and then to derive new results out of the incomplete information these answer sets convey altogether. One reason for this drawback is the local performance of the ASP's negation as failure (NAF) operator (aka, default negation): note that NAF can only reflect incomplete information of each answer

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[1] The use of variables in ASP-programs is understood as abbreviations for the collection of their ground (variable-free) instances. Thus, for simplicity, in this paper we restrict the language of (epistemic) ASP to the propositional case. In ASP, a ground literal is a propositional variable (here, referred to as an *atom*) p or a *strongly-negated* propositional variable $\sim p$.

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set individually, but in order to extend the issue to the whole range of answer sets for global reasoning, we need epistemic modal operators, which are able to quantify over a collection of answer sets.

The first approach of this line of research is Gelfond's *epistemic specifications* (ES_{91}) [8, 9]: he extended ASP with epistemic constructs called *subjective literals*. Indeed, with the inclusion of the epistemic modalities K and M (respectively having the literal readings "*known*" and "*may be believed*" in ES_{91}), he could encode information of answer set collections. The interpretation of this new language was in terms of *world-views*—collections $\mathscr{A}$ of valuations A, each of which constitutes a minimal pointed classical S5-model[2] $(\mathscr{A},A)$ of a program Π w.r.t. truth and knowledge. Similarly to answer sets, world-views are also reduct-based. The reduct definition of the former eliminates default-negated constructs (i.e., NAF) w.r.t. a candidate answer set A so that the reduct is a positive ASP-program, excluding NAF; whereas the goal of the latter in ES_{91} is, in principle, to remove epistemic constructs w.r.t. a candidate world-view $\mathscr{A}$. Thus, the resulting program $\Pi^{\mathscr{A}}$ appears to be a regular ASP-program, possibly including NAF (but excluding K and M). Then, we generate the collection $\mathscr{A}'$ of all answer sets of this reduct $\Pi^{\mathscr{A}}$. Finally, if $\mathscr{A}'$ equals our candidate model $\mathscr{A}$, then we call $\mathscr{A}$ a *world-view* of the original program Π.

Researchers have soon realised that ES_{91} allows unsupported world-views. Then, not only Gelfond himself [10], but also many others have come up with several different semantics proposals for epistemic specifications (ES); one following the other in order to get rid of newly-appearing unintended results. The majority [12, 21, 14, 13, 25, 26] are reduct-based world-view semantics. The rest [29, 23, 28, 4] are inspired by Pearce's equilibrium-model approach [19], characterising answer sets on a purely logical domain through minimal model reasoning. They are based on epistemic extensions of equilibrium logic.

Up to recently, novel formalisms of ES were basically tested in terms of an increasing list of examples where some previous approaches gave unsatisfactory results. However, this informal comparison method started to be confusing as other critical programs were found after each time a new proposal had been suggested. In the end, it appeared that none could provide intended results for the entire list, and worse, some disagreement on the understanding of programs occurred. To that end, Cabalar et al. [4] introduced some formal criteria, that are inherited from ASP, so as to facilitate the search of a successful semantics. Although there are newly-emerging objections [22] to their soundness (even at the ASP level), to us, that was a significant initiative to extend ASP's some well-known structural properties to the epistemic case in order to formally support a possible semantics proposal. We here slightly discuss ASP's possible foundational problems, and accordingly, the validity of these properties. We mainly aim at enhancing ASP's expressivity by epistemic modalities, and while doing so, we basically accept GL's answer sets as our underlying semantics. However, we partly agree that especially the epistemic extensions of such properties are under debate and had better be improved, which is the subject of another work. Briefly, here, we are not in search of a new semantics, compatible with the standards offered by Cabalar et al.

In this paper, we basically make a comprehensive analysis of the previous semantics approaches of ES, revealing their (dis)advantageous points. We think that this search is important to lead the way for a successful semantics. Particularly, we propose reflexive autoepistemic ASP (RAEASP) as an alternative to Cabalar et al.'s recent approach called autoepistemic ASP (AEASP). Thus, we also use Schwarz's [20] minimal model techniques, but propose a formalism closer in spirit to the other approaches because in RAEASP, the epistemic operator K formalises knowledge, while in AEASP, it represents belief. We also extend the well-known propositions of here-and-there logic (HT) to the epistemic case and use them to simplify some complex programs in order to clarify their correct meaning. We also very roughly discuss paracoherent reasoning for epistemic logic programs, similarly to regular ASP-programs [1].

The rest of the paper is organised as follows: Section 2 introduces epistemic specifications (ES) and

[2]Particularly here, we regard S5-models as cluster structures in which every world is related to any other, including itself.

its relatively successful semantics approaches. Section 2.5 proposes a reflexive extension of autoepistemic ASP in order to reason about a rational agent's own knowledge rather than self-belief. Section 3 provides some formal tools, ensuring the reasonable behaviour of epistemic programs: in particular, Section 3.1 recalls formal properties of ES, suggested recently. Section 3.2 provides epistemic extensions of some useful equivalences of HT. Section 3.3 gives a detailed comparison between semantics approaches discussed in the paper by means of examples. Section 4 concludes the paper with future work plan.

2 Background and Related Work

In this section, we introduce epistemic specifications (ES) and the semantics approaches, proposed so far. Since Gelfond's first version, named ES_{91} here, was slightly and successively refined by several authors as ES_{94} [9], ES_{11} [10], ES_{14} [12], ES'_{16} [14], and finally ES_{18} [13], we begin with recalling the latest version: the language of ES ($\mathscr{L}_{\mathsf{ES}}$) comprises four kinds of literals; *objective literals (l)*, *extended objective literals (L)*, *subjective literals (g)*, and *extended subjective literals (G)* as identified below:

l		L		g		G	
p	$\sim p$	l	$\mathtt{not}\,l$	$\mathsf{K}\,l$	$\mathsf{M}\,l$	g	$\mathtt{not}\,g$

where p ranges over an infinite set $\mathbb{P}$ of atoms. $\mathscr{L}_{\mathsf{ES}}$ has 2 negations. *Strong negation*, symbolised by '$\sim$', represents direct and explicit falsity. Weaker *negation as failure* (NAF), denoted by 'not', helps us partly encode incomplete information: $\sim p$ implies $\mathtt{not}\,p$ for an atomic p, but not vice versa. So, if $\mathtt{not}\,p$ holds, then either $\sim p$ is the case (i.e., p is false), or p is assumed false since the truth of p cannot be justified due to lack of evidence. Consequently, while double $\sim$ vanishes, notnot does not. Also note that $\mathtt{not}\,p$ can be defined as a shorthand for $\bot \leftarrow p$, but $\sim p$ is not a shorthand. $\mathtt{not}\,p$ reads "*p is false by default*", and $\mathtt{notnot}\,p$ means "*p is not false, but its truth cannot be guaranteed*". Different from intuitionistic modal logics, in ES, the belief operator M is the dual of the knowledge operator K, i.e., $\mathsf{M} \overset{\text{def}}{=} \mathtt{not}\mathsf{K}\mathtt{not}$.

A *rule* is a logical statement of the form head←body. In particular, a rule r of ES has the structure

$$l_1\,\mathtt{or}\ \ldots\ \mathtt{or}\,l_m \leftarrow e_1\ ,\ \ldots\ ,\ e_n$$

in which $\mathtt{body}(\mathtt{r})$ viz. $e_1, \ldots, e_n$ is made up of arbitrary (i.e., extended objective or extended subjective) literals of ES, and $\mathtt{head}(\mathtt{r})$ viz. $l_1\,\mathtt{or} \ldots \mathtt{or}\,l_m$ is composed of only objective literals. Note that 'or', '←', and ',' respectively represent disjunction, reversed implication and conjunction. When $m = 0$, we suppose $\mathtt{head}(\mathtt{r})$ to be $\bot$ and call the rule r a *constraint* (headless rule). In particular, when $\mathtt{body}(\mathtt{r})$ is composed of exclusively extended subjective literals, we call it a *subjective constraint*. When $n = 0$, we suppose $\mathtt{body}(\mathtt{r})$ to be $\top$ and call r a *fact* (bodiless rule). We usually disregard $\bot$ and $\top$ in such special rules. An *(epistemic) logic program*, abbreviated as (E)LP, is a finite collection of (epistemic) rules.

2.1 Kahl et al.'s semantics approach (ES_{18}): modal reduct w.r.t. a classical S5-model

Given a non-empty collection $\mathscr{A}$ of valuations, let $A \in \mathscr{A}$ be arbitrary. Then, satisfaction of literals is defined as follows: for an objective literal l, an extended objective literal L, and a subjective literal g,

$$\begin{array}{llll} \mathscr{A}, A \models_{\mathsf{ES}} l & \text{if} \quad l \in A; & \mathscr{A}, A \models_{\mathsf{ES}} \mathtt{not}\,l & \text{if} \quad l \notin A. \\ \mathscr{A}, A \models_{\mathsf{ES}} \mathsf{K}L & \text{if} \quad \mathscr{A}, A' \models_{\mathsf{ES}} L \ \text{ for every } A' \in \mathscr{A}; & \mathscr{A}, A \models_{\mathsf{ES}} \mathtt{not}\,g & \text{if} \quad \mathscr{A}, A \not\models_{\mathsf{ES}} g. \\ \mathscr{A}, A \models_{\mathsf{ES}} \mathsf{M}L & \text{if} \quad \mathscr{A}, A' \models_{\mathsf{ES}} L \ \text{ for some } A' \in \mathscr{A}; & & \end{array}$$

Table 1: Kahl et al.'s original definition of reduct, and SE's implicitly inferred reduct definition.

	Original reduct definition of ES_{18}		**Implicit reduct definition of** ES_{16}	
literal G	if $\mathscr{A} \models_{\mathsf{ES}} G$	if $\mathscr{A} \not\models_{\mathsf{ES}} G$	if $\mathscr{A} \models_{\mathsf{ES}} G$	if $\mathscr{A} \not\models_{\mathsf{ES}} G$
$\mathsf{K}l$	**replace by** $\boldsymbol{l}$	replace by $\bot$	**replace by** $\mathbf{\texttt{notnot}}\boldsymbol{l}$	replace by $\bot$
$\mathsf{M}l$	replace by $\top$	replace by $\texttt{notnot}l$	replace by $\top$	replace by $\texttt{notnot}l$
$\texttt{not}\mathsf{K}l$	replace by $\top$	replace by $\texttt{not}l$	replace by $\top$	replace by $\texttt{not}l$
$\texttt{not}\mathsf{M}l$	replace by $\texttt{not}l$	replace by $\bot$	replace by $\texttt{not}l$	replace by $\bot$

Satisfaction of an objective literal l is independent of $\mathscr{A}$, and satisfaction of a subjective literal g is independent of A. So, we can safely write $\mathscr{A} \models_{\mathsf{ES}} g$ or $A \models_{\mathsf{ES}} l$. Satisfaction of an ELP Π is defined by:

$$\mathscr{A},A \models_{\mathsf{ES}} \Pi \quad \text{if} \quad \mathscr{A},A \models_{\mathsf{ES}} \mathtt{r} \;\; (\text{i.e., ``}\mathscr{A},A \models_{\mathsf{ES}} \mathtt{body}(\mathtt{r}) \;\; \text{implies} \;\; \mathscr{A},A \models_{\mathsf{ES}} \mathtt{head}(\mathtt{r})'')$$

for every rule $\mathtt{r} \in \Pi$. When $\mathscr{A},A \models_{\mathsf{ES}} \Pi$ for every $A \in \mathscr{A}$, we say that $\mathscr{A}$ is a classical S5-model of Π. In order to decide if $\mathscr{A}$ is further a world-view of Π, we first compute the (modal) reduct $\Pi^{\mathscr{A}} = \{\mathtt{r}^{\mathscr{A}} : \mathtt{r} \in \Pi\}$ of Π w.r.t. $\mathscr{A}$, where we eliminate the modal operators K and M according to Table 1. Therefore, $\Pi^{\mathscr{A}}$ is a regular (nonepistemic) ASP-program. Then, we generate the set $\mathtt{Ep}(\Pi)$ of *epistemic negations* (literals having the form of $\texttt{not}\mathsf{K}\,l$ or $\mathsf{M}\,l$) of Π by transforming each extended subjective literal appearing in Π into one of these sorts. As an illustration, $\mathtt{Ep}(\Pi') = \{\texttt{not}\mathsf{K}p, \mathsf{M}q, \texttt{not}\mathsf{K}s, \mathsf{M}t\}$ for the program $\Pi' = \{t \leftarrow \mathsf{K}p, \mathsf{M}q, \texttt{not}\mathsf{K}s, \texttt{not}\mathsf{M}t\}$. Next, we take the elements of $\mathtt{Ep}(\Pi)$, satisfied by $\mathscr{A}$ and form the set $\mathtt{Ep}(\Pi)|_{\mathscr{A}} = \{G \in \mathtt{Ep}(\Pi) : \mathscr{A} \models_{\mathsf{ES}} G\}$. Finally, $\mathscr{A}$ is a *world-view* of Π if[3] $\mathscr{A} \stackrel{\mathsf{fp}}{=} \mathtt{AS}(\Pi^{\mathscr{A}})$, and

> there is no classical S5-model $\mathscr{A}'$ of Π such that $\mathscr{A}' \stackrel{\mathsf{fp}}{=} \mathtt{AS}(\Pi^{\mathscr{A}'})$ and
> (*knowledge-minimisation property w.r.t. epistemic negation*) $\mathtt{Ep}(\Pi)|_{\mathscr{A}'} \supset \mathtt{Ep}(\Pi)|_{\mathscr{A}}$

where $\mathtt{AS}(\Pi)$ refers to the set of all answer sets of a nonepistemic program Π. However, knowledge-minimisation w.r.t. $\mathtt{Ep}(\Pi)$ may suggest an ambiguity when Π's classical S5-models $\mathscr{A}_1$ and $\mathscr{A}_2$, satisfying $\mathscr{A}_1 \stackrel{\mathsf{fp}}{=} \mathtt{AS}(\Pi^{\mathscr{A}_1})$ and $\mathscr{A}_2 \stackrel{\mathsf{fp}}{=} \mathtt{AS}(\Pi^{\mathscr{A}_2})$, give rise to $\left|\mathtt{Ep}(\Pi)|_{\mathscr{A}_1}\right| \neq \left|\mathtt{Ep}(\Pi)|_{\mathscr{A}_2}\right|$, but $\mathtt{Ep}(\Pi)|_{\mathscr{A}_1}$ and $\mathtt{Ep}(\Pi)|_{\mathscr{A}_2}$ are not comparable w.r.t. subset relation [13]: for such $\mathscr{A}_1$ and $\mathscr{A}_2$, it is potential to have, for instance, $\mathtt{Ep}(\Pi)|_{\mathscr{A}_1} = \{\texttt{not}\mathsf{K}p, \texttt{not}\mathsf{K}q\}$ and $\mathtt{Ep}(\Pi)|_{\mathscr{A}_2} = \{\texttt{not}\mathsf{K}s\}$. So, both $\mathscr{A}_1$ and $\mathscr{A}_2$ are world-views of Π while $\mathscr{A}_1$ makes more atoms unknown, compared to $\mathscr{A}_2$. Another point is that we do not follow a similar truth-minimisation attitude for $\mathtt{NAF}$ in ASP, e.g., $\mathtt{AS}(p\,\texttt{or}\,\texttt{not}p) = \{\{p\}, \emptyset\}$. While we have $\emptyset \models \texttt{not}p$ and $\{p\} \not\models \texttt{not}p$ for the unique default-negated atom $\texttt{not}p$, we do not prefer $\emptyset$ rather than $\{p\}$ as it minimises truth "more" than $\{p\}$. Hence, to us, knowledge-minimality per $\mathtt{Ep}(\Pi)$ had better be revised.

The main contribution of ES_{18} over its pioneer ES'_{16} as a final follow-up is *world-view constructs*: ES_{18} introduces the symbol $\stackrel{\mathsf{wv}}{\leftarrow}$ which reads "it is not a world-view if". This gives us a chance to transform subjective constraints $\leftarrow G_1, \ldots, G_n$ into $\stackrel{\mathsf{wv}}{\leftarrow} G_1, \ldots, G_n$ so that they perform analogously to how constraints affect answer-sets in ASP: they (at most) rule out world-views, violating them. Note that the semantics of ES'_{16} has lost this property while trying to guarantee intended results for certain other programs.

[3] The fixed point equation $\stackrel{\mathsf{fp}}{=}$ is basically to ensure stability of truth-minimisation, but, in essence, it also accommodates kind of knowledge-minimisation: e.g., given the rule $p\,\texttt{or}\,q$, it only holds for $\{\{p\},\{q\}\}$; yet, it does not hold for $\{\{p\}\}$ or $\{\{q\}\}$.

2.2 Shen&Eiter's approach (ES_{16}): modal reduct w.r.t. a set of epistemic negations

Another reduct-based semantics for ES has been proposed by Shen and Eiter (SE) [21]: given an ELP Π, let $\mathscr{A}$ be its classical S5-model, and let $\mathtt{Ep}(\Pi)|_{\mathscr{A}}$ be the set of all its epistemic negations, satisfied by $\mathscr{A}$ (see Sect. 2.1). We first transform Π into its reduct $\Pi^{\mathtt{Ep}(\Pi)|_{\mathscr{A}}}$ w.r.t. $\mathtt{Ep}(\Pi)|_{\mathscr{A}}$ by replacing every $G \in \mathtt{Ep}(\Pi)|_{\mathscr{A}}$ with $\top$, and every $G \in \mathtt{Ep}(\Pi) \setminus \mathtt{Ep}(\Pi)|_{\mathscr{A}}$ with $\mathtt{not}l$ if $G{=}\mathtt{not}\mathsf{K}l$ and with $\mathtt{notnot}l$ if $G{=}\mathsf{M}l$. Then, $\mathscr{A}$ is a *world-view* of Π if $\mathscr{A} \overset{\mathsf{fp}}{=} \mathtt{AS}(\Pi^{\mathtt{Ep}(\Pi)|_{\mathscr{A}}})$, and there is no classical S5-model $\mathscr{A}'$ of Π such that $\mathscr{A}' \overset{\mathsf{fp}}{=} \mathtt{AS}(\Pi^{\mathtt{Ep}(\Pi)|_{\mathscr{A}'}})$ and $\mathtt{Ep}(\Pi)|_{\mathscr{A}'} \supset \mathtt{Ep}(\Pi)|_{\mathscr{A}}$. Clearly, the reduct definitions are where ES_{18} and ES_{16} only differ. However, as Table 1 shows above, it is possible to arrange an equivalent version of SE's reduct definition, and this allows us to compare the approaches of ES_{18} and ES_{16} more easily.

Note that $\mathtt{Ep}(\Pi)$ includes all extended subjective literals of Π to be taken into the reduct transformation of ES_{18}, but as encoded in the form of an epistemic negation. So, given a candidate world-view $\mathscr{A}$ and a subjective literal $\mathsf{K}l$ appearing in Π (but not in the scope of $\mathtt{NAF}$), assume that $\mathscr{A} \models_{\mathsf{ES}} \mathsf{K}l$. Note that $\mathtt{Ep}(\Pi)$ contains $\mathsf{K}l$ in the form of $\mathtt{not}\mathsf{K}l$, and $\mathtt{not}\mathsf{K}l \notin \mathtt{Ep}(\Pi)|_{\mathscr{A}}$ since $\mathscr{A} \not\models_{\mathsf{ES}} \mathtt{not}\mathsf{K}l$. As a result, $\mathtt{not}\mathsf{K}l$ is transformed into $\mathtt{not}l$ w.r.t. SE's reduct definition; yet the literal appears as $\mathsf{K}l$ in the program Π. SE considers $\mathsf{K}l$ and $\mathtt{notnot}\mathsf{K}l$ to be equivalent, so since they can transform $\mathtt{not}(\mathtt{not}\mathsf{K}l)$ into $\mathtt{not}(\mathtt{not}l)$, they also accept the reduct of $\mathsf{K}l$ into $\mathtt{notnot}l$ to be legitimate. Moreover, in their original definition, $\mathtt{notnot}l$ is reduced to l in this case. To sum up, when $\mathscr{A} \models_{\mathsf{ES}} \mathsf{K}l$, the SE-reduct transforms $\mathsf{K}l$ into l. While the other cases are reasonable, this case is not cogent for us. There are two problematic issues here: first, the original language of ES_{16} does not contain the modal operators as primitives, instead it has three negations; $\sim$, $\mathtt{not}$, and $\mathtt{NOT}$, where the last denotes epistemic negation $\mathtt{not}\mathsf{K}$. Thus, K and M exist as derived operators respectively in the form of $\mathtt{notNOT}$ and $\mathtt{NOTnot}$. Such derivations use the equivalence between $\mathsf{K}l$ and $\mathtt{notnot}\mathsf{K}l$. In our opinion, $\mathsf{K}l$ and $\mathtt{notnot}\mathsf{K}l$ are classically equivalent, similarly to the ASP-literals, l and $\mathtt{notnot}l$; yet, they cannot be considered strongly equivalent, allowing above transitions. In one sense, SE's language includes $\mathtt{notnot}\mathsf{K}l$ instead of $\mathsf{K}l$, and there is no formal way to produce $\mathsf{K}l$ as a derived formula. Second, while it is questionable to reduce $\mathtt{notnot}l$ to l while taking the reduct of $\mathsf{K}l$, replacing $\mathsf{K}l$ by $\mathtt{notnot}l$ in the reduct definition of ES_{16} is probably harder to accept. Generally speaking, taking the reduct of a positive construct $\mathsf{K}l$ may be dangerous. We discuss the issue in [25] and propose an alternative reduct definition of ES, oriented only to remove $\mathtt{NAF}$, aligning with the approach of ASP. In particular, we do not take the reduct of $\mathsf{K}l$. To sum up, although ES_{18} and ES_{16} look different, they are similar structurally and give the same results under SE's original reduct definition.

The following semantics for ELPs are a lot different from the reduct-based approaches, mentioned above. They are defined on a purely logical domain as extensions of equilibrium models[4].

2.3 Fariñas et al.'s approach (ES_{15}): autoepistemic equilibrium models (AEEMs)

Here-and-there logic (HT) is a 3-valued monotonic logic, which is intermediate between classical logic and intuitionistic logic. An HT-model is an ordered pair (H,T) of valuations $H,T \subseteq \mathbb{P}$, satisfying $H \subseteq T$. Equilibrium logic (EL) is a general purpose nonmonotonic formalism, whose semantics is based on a truth-minimality condition over HT-models. Pearce [19] basically proposed EL in order to provide a purely logical foundation of ASP. Inspired by its success as ASP's general framework, Fariñas et al. [23, 6, 28] introduced an epistemic extension of EL, named ES_{15} here, in order to suggest an alternative semantics not only for ES, but also for nested ELPs. This section briefly recalls the approach of ES_{15}.

[4] A first step towards epistemic equilibrium logic belongs to [29], which embeds ES_{94}, but ES_{94} is obselete today.

2.3.1 Epistemic here-and-there logic (EHT) and its equilibrium models

EHT extends HT with nondual epistemic modalities K and $\hat{\mathsf{K}}$: both operators are primitive; while K is identical to $\mathsf{K}\in\mathscr{L}_{\mathsf{ES}}$, the belief operator $\hat{\mathsf{K}}$ (read "*believed*") is so different from $\mathsf{M}\in\mathscr{L}_{\mathsf{ES}}$. This is justified by the fact that M is derived as notKnot in ES and so translated to EHT as $\neg\mathsf{K}\neg$ where $\neg$ refers to EHT-negation. As will be shown later in Sect. 3.3, $\neg\mathsf{K}\neg\varphi$, $\neg\neg\hat{\mathsf{K}}\varphi$, and $\hat{\mathsf{K}}\neg\neg\varphi$ are all equivalent in EHT. Thus, $\mathsf{M}\in\mathscr{L}_{\mathsf{ES}}$ corresponds to notnot$\hat{\mathsf{K}}$ or $\hat{\mathsf{K}}$notnot in an extension of ES with $\hat{\mathsf{K}}$. Notice that the difference between $\mathsf{M}p$ and $\hat{\mathsf{K}}p$ in ES resembles that of notnot$p$ and p in ASP. As a result, in an extended language, we expect $\mathsf{M}p$ not to have a world-view, whereas $\{\emptyset,\{p\}\}$ is one understandable world-view for $\hat{\mathsf{K}}p$. The language of EHT ($\mathscr{L}_{\mathsf{EHT}}$) is given by the following grammar: for $p\in\mathbb{P}$,

$$\varphi ::= p \mid \bot \mid \varphi\wedge\varphi \mid \varphi\vee\varphi \mid \varphi\rightarrow\varphi \mid \mathsf{K}\varphi \mid \hat{\mathsf{K}}\varphi.$$

As usual, $\neg\varphi$, $\top$, and $\varphi\leftrightarrow\psi$ respectively abbreviate $\varphi\rightarrow\bot$, $\bot\rightarrow\bot$, and $(\varphi\rightarrow\psi)\wedge(\psi\rightarrow\varphi)$. A theory is a finite set of formulas. An ELP Π is translated into the corresponding EHT-theory Π^* via a map $(.)^*$: given a prototypical program $\Pi=\{r_1,r_2\}$ where $r_1 = p\,\texttt{or}\,{\sim}q\leftarrow \mathsf{M}r, \texttt{not}s$ and $r_2 = q\leftarrow\texttt{notK}p$, we have:

$$\Pi^* = \big((\neg\mathsf{K}\neg r\wedge\neg s)\rightarrow(p\vee\widetilde{q})\big)\ \wedge\ \big(\neg\mathsf{K}p\rightarrow q\big)\ \wedge\ \neg\big(q\wedge\widetilde{q}\big)$$

where ${\sim}q$ is evaluated as a new atom $\widetilde{q}\in\mathbb{P}$, entailing the formula $\neg\big(q\wedge\widetilde{q}\big)$ to be inserted into Π^*.

An EHT-model $\langle\mathscr{A},\mathtt{s}\rangle$ is a refinement of a classical S5-model $\mathscr{A}$ in which valuations $A\in\mathscr{A}$ are replaced by HT-models $(\mathtt{s}(A),A)$ w.r.t. a function $\mathtt{s}:\mathscr{A}\rightarrow 2^{\mathbb{P}}$, assigning to each $A\in\mathscr{A}$ one of its subsets, i.e., $\mathtt{s}(A)\subseteq A$. We call s a *subset* function. Thus, $\langle\mathscr{A},\mathtt{s}\rangle$ is represented explicitly by $\big\{\big(\mathtt{s}(A),A\big)\big\}_{A\in\mathscr{A}}$. Satisfaction of a formula $\varphi\in\mathscr{L}_{\mathsf{EHT}}$ is defined recursively w.r.t. to the following truth conditions:

$\langle\mathscr{A},\mathtt{s}\rangle,A\models_{\mathsf{EHT}} p$	if	$p\in\mathtt{s}(A)$;
$\langle\mathscr{A},\mathtt{s}\rangle,A\models_{\mathsf{EHT}} \varphi\rightarrow\psi$	if	$\big(\langle\mathscr{A},\mathtt{s}\rangle,A\not\models_{\mathsf{EHT}}\varphi$ or $\langle\mathscr{A},\mathtt{s}\rangle,A\models_{\mathsf{EHT}}\psi\big)$ and $\big(\langle\mathscr{A},id\rangle,A\not\models_{\mathsf{EHT}}\varphi$ or $\langle\mathscr{A},id\rangle,A\models_{\mathsf{EHT}}\psi\big)$;
$\langle\mathscr{A},\mathtt{s}\rangle,A\models_{\mathsf{EHT}} \mathsf{K}\varphi$	if	$\langle\mathscr{A},\mathtt{s}\rangle,A'\models_{\mathsf{EHT}}\varphi$ for every $A'\in\mathscr{A}$;
$\langle\mathscr{A},\mathtt{s}\rangle,A\models_{\mathsf{EHT}} \hat{\mathsf{K}}\varphi$	if	$\langle\mathscr{A},\mathtt{s}\rangle,A'\models_{\mathsf{EHT}}\varphi$ for some $A'\in\mathscr{A}$;

where *id* denotes the identity function. Those of $\bot$, $\wedge$ and $\vee$ are standard. The EHT-model $\langle\mathscr{A},id\rangle$ is called *total* and identical to the classical S5-model $\mathscr{A}$. Then, $\mathscr{A}$ is an *epistemic equilibrium model* (EEM) of $\varphi\in\mathscr{L}_{\mathsf{EHT}}$ if $\mathscr{A}$ is a classical S5-model φ and satisfies the following truth-minimality condition:

$$\text{for every possible subset function } \mathtt{s} \text{ on } \mathscr{A} \text{ with } \mathtt{s}\neq id, \text{ there is } A\in\mathscr{A} \text{ s.t. } \langle\mathscr{A},\mathtt{s}\rangle,A\not\models_{\mathsf{EHT}}\varphi. \quad (1)$$

EEMs can only minimise truth (similarly to that of EL). They do not involve a knowledge-minimisation criterion. So, the EEM approach may bring out undesired results, especially in the presence of disjunction. To overcome this problem, ES_{15} uses a *selection process* over EEMs by comparing them with each other according to set inclusion $\subseteq$, and a φ-indexed preorder $\leq_\varphi$ defined as follows: for $\mathscr{A},\mathscr{A}'\in\mathtt{EEM}(\varphi)$,

$$\mathscr{A}\leq_\varphi\mathscr{A}' \quad\text{iff}\quad \text{for every } A_0\in\bigcup\mathtt{EEM}(\varphi),\ \text{if } \mathscr{A}\cup\{A_0\},\mathscr{A}\models^*\varphi \text{ then } \mathscr{A}'\cup\{A_0\},\mathscr{A}'\models^*\varphi$$

where $\mathtt{EEM}(\varphi)$ denotes the set of all EEMs of φ, and $\bigcup\mathtt{EEM}(\varphi)$ is their union. Moreover[5], $\mathscr{A}\cup\{A_0\},\mathscr{A}\models^*\varphi$ means $\mathscr{A}\cup\{A_0\},A\models_{\mathsf{S5}}\varphi$ for every $A\in\mathscr{A}$, and $\langle\mathscr{A}\cup\{A_0\},\mathtt{s}\rangle,\mathscr{A}\not\models_{\mathsf{EHT}}\varphi$ for every $\mathtt{s}\neq id$ such that $\mathtt{s}(A_0)=A_0$. Then, the strict version of $\leq_\varphi$ is standard: $\mathscr{A}<_\varphi\mathscr{A}'$ if $\mathscr{A}\leq_\varphi\mathscr{A}'$ and $\mathscr{A}\not\leq_\varphi\mathscr{A}'$. An *autoepistemic equilibrium model* (AEEM) of φ is the maximal EEM of φ w.r.t. these orderings. However,

[5]Given $\mathscr{A}\subseteq\mathscr{B}$, the pair $(\mathscr{B},\mathscr{A})$ denotes a multipointed S5-model where each $A\in\mathscr{A}$ is a designated (actual) world. Similarly, $(\langle\mathscr{B},\mathtt{s}\rangle,\mathscr{A})$ denotes a multipointed EHT-model where $\langle\mathscr{A},\mathtt{s}\rangle$ is the collection of designated HT-models of $\langle\mathscr{B},\mathtt{s}\rangle$.

choosing AEEMs w.r.t. simultaneously performing two orderings may be dangerous. So, ES_{15} should guarantee via a formal proof that these orderings do not contradict each other because it seems possible, in principle, to have $\mathscr{A}_1, \mathscr{A}_2 \in \mathtt{EEM}(\varphi)$, satisfying both $\mathscr{A}_1 \subset \mathscr{A}_2$ and $\mathscr{A}_2 <_\varphi \mathscr{A}_1$. Moreover, the definition of $\leq_\varphi$ is too heavy to grasp the intuition behind. While the preorder $\leq_\varphi$ gets inspiration from Moore's autoepistemic logic [17] and Levesque's all-that-I-know logic [15], it does not use the exact techniques of these formalisms to maximise ignorance. Instead, ES_{15} checks its candidate S5-models $\mathscr{A}_1, \mathscr{A}_2 \in \mathtt{EEM}(\varphi)$ in doubles by first enlarging them with a possible world A_0 appearing in some model of $\mathtt{EEM}(\varphi)$ and then comparing their behaviour relative to φ. Note that while testing them, if the enlarged model $(\mathscr{A}_1 \cup \{A_0\}, \mathscr{A}_1)$ is a multipointed EEM of φ, then this is an advantage for $\mathscr{A}_1$ on the way to jump the maximality test, but it also means that $\mathscr{A}_1$ is not stable w.r.t. knowledge in one sense. Thus, while this tool eliminates undesired models in many cases, it does not fulfill the requirement of being understandable in my opinion and appears a bit ad hoc. Still, ES_{15} is the first formalism that has provided a "standard" epistemic extension of EL and together with [29], leads the way to more successful follow-ups such as ES_{20}. The following section introduces Cabalar et al.'s recent semantics proposal called ES_{20}.

2.4 Cabalar et al.'s approach (ES_{20}): founded autoepistemic equilibrium models

Autoepistemic logic (AEL) [17] is one of the major types of nonmonotonic reasoning, allowing a rational agent to reason about her own beliefs. Inspired by AEL[6], ES_{15} adds a valuation to EEMs and examines the behavior of augmented models to determine AEEMs. However, this method does not coincide with $\mathsf{KD45}$'s minimal-model techniques because the AEEM-selection process takes place in an S5-setting. From this respect, Cabalar et al.'s approach [4], named ES_{20} here, is the first to formally combine EL and AEL with the purpose of inserting the introspective reasoning of the latter into the former. To distinguish the similar concepts of ES_{15} and ES_{20}, when necessary, we respectively add the subscripts 15 and 20.

The language $\mathscr{L}_{\mathsf{EHT}_{20}}$ is a fragment of $\mathscr{L}_{\mathsf{EHT}_{15}}$, excluding $\hat{\mathsf{K}}$, but also $\mathsf{K}\varphi$ reads differently: *φ is the agent's belief*. Semantically, it is straightforward to extend EHT_{20} with $\hat{\mathsf{K}}$, but its meaning is not obvious.

There are two important differences of $\mathtt{EHT}_{20}$-models from functional $\mathtt{EHT}_{15}$-models defined above:

First, $\mathtt{EHT}_{20}$-models are almost the same as *relational* $\mathtt{EHT}_{15}$-models (see [28], Sect. 8) when we consider them simply as nonempty collections of arbitrary HT-models, but disregard the relations between these HT-models. Probably, the only (negligible) difference is that the latter can be formed as a multiset of HT-models. In order to achieve this, instead of a subset function $\mathtt{s}$, EHT_{20} employs a serial subset relation (i.e., a multivalued subset function) $\mathtt{s_r}$, relating each $A \in \mathscr{A}$ to at least one element from 2^A. So, using the S5-model $\mathscr{A}$ and $\mathtt{s_r}$, we can produce the HT-model collections $\{(H,A) : H\mathtt{s_r}A\}_{A\in\mathscr{A}}$. For instance, while the S5-model $\{A\}$, for $A = \{p,q\}$, can give rise to the functional $\mathtt{EHT}_{15}$-models $\{(\emptyset,A)\}$, $\{(\{p\},A)\}$, $\{(\{q\},A)\}$, and $\{(A,A)\}$, in EHT_{20}, we can additionally obtain the following nontotal $\mathtt{EHT}_{20}$-models $\{(\{p\},A),(\{q\},A)\}$, $\{(\emptyset,A),(\{p\},A),(\{q\},A)\}$, $\{(\emptyset,A),(A,A)\}$, etc. We represent $\mathtt{EHT}_{20}$-models with a similar notation $(\mathscr{A},\mathtt{s_r})$ where $\mathtt{s_r}$ refers to a multivalued subset function on a domain $\mathscr{A}$.

Second, $\mathtt{EHT}_{20}$-models are in the form of $\mathsf{KD45}$-models, while $\mathtt{EHT}_{15}$-models are special S5-models. Given nonempty collections $\mathscr{A},\mathscr{B} \subseteq 2^{\mathbb{P}}$ of valuations with $\mathscr{A} \subseteq \mathscr{B}$ and a multivalued subset function $\mathtt{s_r}$ defined on a domain $\mathscr{B}$, a $\mathsf{KD45}$-model $\langle \mathscr{B}, \mathtt{s_r} \rangle$ is a weaker form of an S5-model $\langle \mathscr{A}, \mathtt{s_r}|_{\mathscr{A}} \rangle$ as it may contain an additional world $(\mathtt{s_r}(B),B)$ for $B \notin \mathscr{A}$, outside the maximal-cluster structure $\langle \mathscr{A}, \mathtt{s_r}|_{\mathscr{A}} \rangle$. Note that $\mathtt{s_r}|_{\{B\}}$ is an ordinary (singlevalued) subset function. Furthermore, while $(\mathtt{s_r}(B),B)$ relates exclusively to all worlds of the maximal-cluster $\langle \mathscr{A}, \mathtt{s_r}|_{\mathscr{A}} \rangle$ and so is irreflexive, no world in $\langle \mathscr{A}, \mathtt{s_r}|_{\mathscr{A}} \rangle$ can relate to $(\mathtt{s_r}(B),B)$. In other words, an $\mathtt{EHT}_{20}$-model is a refinement of a classical $\mathsf{KD45}$-model, whose

[6] Schwarz [20] showed that the nonmonotonic extensions of modal logic $\mathsf{KD45}$ and modal logic $\mathsf{SW5}$ under the minimal-model semantics respectively correspond to AEL and reflexive AEL (RAEL), interpreted by stable expansions.

valuations are replaced by HT-models w.r.t. the multivalued subset function $\mathtt{s_r}$. Hence, when $\mathtt{s_r} = id$, $\langle \mathscr{B}, \mathtt{s_r} \rangle$ corresponds to the classical KD45-model $\mathscr{B}$. When $\mathscr{A} \subset \mathscr{B}$ where $\mathscr{A}$ is a maximal cluster, we say that $\langle \mathscr{B}, id \rangle$ is a *proper KD45-extension* of $\langle \mathscr{A}, id \rangle$. Truth conditions of EHT_{20} only differ from those of EHT_{15} for $\mathsf{K}\varphi$ and $\hat{\mathsf{K}}\varphi$ at the world $(\mathtt{s_r}(B), B)$: (in an explicit representation, we underline the world $(\mathtt{s_r}(B), B)$ in the $\mathtt{EHT}_{20}$-model $\langle \mathscr{B}, \mathtt{s_r} \rangle$ to separate it from the elements of the maximal cluster $\langle \mathscr{A}, \mathtt{s_r}|_{\mathscr{A}} \rangle$.)

$$\langle \mathscr{B}, \mathtt{s_r} \rangle, B \models_{\mathsf{EHT}_{20}} \mathsf{K}\varphi \quad \text{if} \quad (\mathscr{A}, \mathtt{s_r}|_{\mathscr{A}}), A \models_{\mathsf{EHT}_{20}} \varphi \;\; \text{for every } A \in \mathscr{A};$$
$$\langle \mathscr{B}, \mathtt{s_r} \rangle, B \models_{\mathsf{EHT}_{20}} \hat{\mathsf{K}}\varphi \quad \text{if} \quad (\mathscr{A}, \mathtt{s_r}|_{\mathscr{A}}), A \models_{\mathsf{EHT}_{20}} \varphi \;\; \text{for some } A \in \mathscr{A}.$$

Notice that since $\mathtt{s_r}$ is a multivalued function on the domain $\mathscr{A}$, the designated world A in the above compact representation of the (pointed) $\mathtt{EHT}_{20}$-model $(\langle \mathscr{A}, \mathtt{s_r}|_{\mathscr{A}} \rangle, A)$ is regarded as a shorthand for all possible HT-models $(H, A) \in \mathtt{s_r}$. The truth-minimality condition of ES_{20} is so more restricted than that of ES_{15} (see 1): for every possible multivalued subset function $\mathtt{s_r}$ on the domain $\mathscr{B}$ satisfying $\mathtt{s_r} \neq id$,

$$\text{there exists } T \in \mathscr{B} \text{ such that } \langle \mathscr{B}, \mathtt{s_r} \rangle, T \not\models_{\mathsf{EHT}} \varphi \tag{2}$$

which amounts to saying that φ is not satisfied at the world (H, T) where $H \mathtt{s_r} T$ in an explicit representation of the model $\langle \mathscr{B}, \mathtt{s_r} \rangle$. To distinguish the similar definitions, we call the condition (2) *relational* truth-minimality and the condition (1) f*unctional* truth-minimality. Then, an *epistemic equilibrium model* ($\mathtt{EEM}_{20}$) of $\varphi \in \mathscr{L}_{\mathsf{EHT}}$ is its classical KD45-model satisfying the truth-minimality condition (2). Thus, when we restrict $\mathtt{EEM}_{20}(\varphi)$ to S5-models, $\mathtt{EEM}_{15}(\varphi)$ is a superset of $\mathtt{EEM}_{20}(\varphi)$ as the former has a more tolerant truth-minimality condition. However, in general, they are incomparable since the latter may additionally include members in the KD45-model form, still remember that world-views are S5-models. Finally, to guarantee knowledge-minimisation, ES_{20} selects S5-models in $\mathtt{EEM}_{20}(\varphi)$, which has no proper KD45-extension in $\mathtt{EEM}_{20}(\varphi)$ and calls them *(founded) autoepistemic equilibrium models*[7] ($\mathtt{AEEM}_{20}$) of $\varphi \in \mathscr{L}_{\mathsf{EHT}}$.

2.5 Our slightly new approach (ES_{21}): reflexive autoepistemic equilibrium models

Modal logic SW5 is a reflexive closure of the modal logic KD45 [24, 27]. Schwarz proposed RAEL (aka, nonmonotonic SW5 under the minimal-model semantics) as an alternative to AEL in a way that it has AEL's all attractive properties. Differently, RAEL defines the modality K so as to model *knowledge* (which limits cyclic arguments) rather than self-belief (which allows them) as in AEL. Moreover, [16] discusses that RAEL captures the default reasoning of ASP much better than AEL. Thus, ES_{20} requires a more thorough analysis for the choice of KD45 rather than SW5 to ensure knowledge-minimisation. This section addresses this issue and presents *reflexive autoepistemic equilibrium models* (RAEEMs).

We first describe the underlying base of the new formalism ES_{21}. Similarly to EHT_{20}, HT and SW5 are incorporated into a monotonic formalism, referred to as EHT_{21} hereafter. The only difference of an $\mathtt{EHT}_{21}$-model from an $\mathtt{EHT}_{20}$-model is that now any HT-model (H, T) in the collection is reflexive, i.e., every such (H, T) can see (access) its own information. Relatedly, an $\mathtt{EHT}_{21}$-model $\langle \mathscr{A}, \mathtt{s_r} \rangle$ is formed from an SW5-model by modifying its classical models (valuations) with HT-models. When $\langle \mathscr{A}, \mathtt{s_r} \rangle$ is total, i.e., $\mathtt{s_r}$ equals the identity function id, we identify the $\mathtt{EHT}_{21}$-model $\langle \mathscr{A}, \mathtt{s_r} \rangle$ with the classical SW5-model $\mathscr{A}$. As a result, different from EHT_{20}, $\mathsf{K}\varphi \rightarrow \varphi$ (*reflexivity*) is an axiom of EHT_{21}. The proper[8] SW5-extension of a maximal-cluster to an SW5-model is defined straightforwardly. Given that $\mathscr{B}$ is a proper SW5-extension of a cluster $\mathscr{A}$, viz. $\mathscr{B}$ is not a cluster, truth conditions of EHT_{21} only vary from those of EHT_{20} for $\mathsf{K}\varphi$ and $\hat{\mathsf{K}}\varphi$ at $(\mathtt{s}(B), B)$ for $B \in \mathscr{B} \setminus \mathscr{A}$, located outside the maximal cluster $\langle \mathscr{A}, \mathtt{s}|_{\mathscr{A}} \rangle$.

$$\langle \mathscr{B}, \mathtt{s} \rangle, B \models_{\mathsf{EHT}_{21}} \mathsf{K}\varphi \quad \text{if} \quad (\mathscr{B}, \mathtt{s}), T \models_{\mathsf{EHT}_{21}} \varphi \;\; \text{for every } T \in \mathscr{B};$$
$$\langle \mathscr{B}, \mathtt{s} \rangle, B \models_{\mathsf{EHT}_{21}} \hat{\mathsf{K}}\varphi \quad \text{if} \quad (\mathscr{B}, \mathtt{s}), T \models_{\mathsf{EHT}_{21}} \varphi \;\; \text{for some } T \in \mathscr{B}.$$

[7] We describe the special models of the ES_{20}-semantics in a slightly different but equivalent way for ease of comparison.

[8] Extending a cluster $\langle \mathscr{A}, \mathtt{s_r} \rangle$ to an SW5-model with an HT-model, already existing in $\langle \mathscr{A}, \mathtt{s_r} \rangle$ does not affect satisfaction.

The definition of (A)EEM is adjusted to the SW5-setting straightforwardly: an *epistemic equilibrium model* ($\mathtt{EEM}_{21}$) of $\varphi \in \mathscr{L}_{\mathsf{EHT}}$ is the classical SW5-model $\mathscr{A}$ of φ, satisfying the truth-minimality condition (2), when viewed as a total EHT_{21}-model $\langle \mathscr{A}, id \rangle$. Similarly to ES_{20}, to minimise knowledge (in other words, to maximise ignorance), ES_{21} also selects S5-models of $\mathtt{EEM}_{21}(\varphi)$, which has no proper SW5-extension in $\mathtt{EEM}_{21}(\varphi)$ and calls them *reflexive autoepistemic equilibrium models* (AEEM21) of φ.

3 Some formal tools towards a well-formed epistemic extension of ASP

This section first recalls the fundamental principles of ES, which are still under question. Then, we demonstrate some validities of EHT_{15} that will be useful for deciding understandable models of ELPs.

3.1 Foundational properties of ES establishing a formal base for successful semantics

Since its introduction in 1991, plenty of semantics proposals have emerged for ES. However, debates and struggles to overcome unintended results still continue. This shows that finding a satisfactory semantics of ES is a challenging task, and therefore, as first realised by Cabalar et al., we need some formal support so as to reveal understandable results and wipe out undesired ones. To this end, they proposed epistemic splitting property (ESP), subjective constraint monotonicity (SCM), foundedness property (FP), supra-ASP, and supra-S5. Expectedly, ES_{20} is compatible with all these properties, whereas each ES_{X} for $x \in \{15, 16, 18\}$, satisfies the last two only. We do not reproduce the definitions here due to space restrictions, and the reader is referred to [4]. Some researchers come up with opposing arguments against their robustness [22], so a thorough examination of these tools is left to another paper. We here check their solidity only roughly, and before doing so, we introduce these principles shortly and informally.

ESP allows for a kind of modularity that guarantees a reasonable behaviour of programs whose subjective literals are stratified. The idea is to separate a program Π into two disjoint subprograms (if possible), *top* and *bottom*, such that top queries bottom through its subjective literals, and bottom never refers to the objective literals of top. If splitting is the case w.r.t. a set U of literals (called *splitting set*), then we calculate the world-views of Π in four steps: first, we compute the world-views $\mathscr{A}_b$ of bottom; second, for each $\mathscr{A}_b$, we take a kind of partial reduct $\Pi_U^{\mathscr{A}_b}$ by replacing the subjective literals g (whose literals are included in U) of top with their truth values in $\mathscr{A}_b$ (i.e., $\top$ if $\mathscr{A} \models_{\mathsf{ES}} g$; $\bot$ otherwise); third, we find the world-views $\mathscr{A}_t$ of $\Pi_U^{\mathscr{A}_b}$ and end with a solution $\langle \mathscr{A}_b, \mathscr{A}_t \rangle$ for Π; finally, we concatenate the components of $\langle \mathscr{A}_b, \mathscr{A}_t \rangle$ in a specific way, resulting in the world-views of the original program Π.

SCM is a special case of ESP and regulates the functioning of subjective constraints: when a subjective constraint $\mathtt{r}$ is added to a program Π, it at most rules out the world-views of Π, but never generates new solutions, i.e., $\Pi \cup \{\mathtt{r}\}$ cannot have a world-view $\mathscr{A}$, where $\mathscr{A}$ is not a world-view of Π per SCM.

FP provides a derivability condition, ensuring self-supported world-views of a program to be rejected.

Supra-ASP means that the unique world-view of a (nonepistemic) regular ASP program Π is the set of all its answer sets, if they exist; otherwise, Π has no world-views. Supra-S5 says that any world-view of an epistemic logic program is an S5-model. Below is an example, illustrating them all.

Example 1 (discussed by Cabalar et al. [4] and Shen&Eiter [22] with opposing claims)
Let $\Psi = \{\mathtt{r}_1, \mathtt{r}_2, \mathtt{r}_3\}$ and $C = \{\mathtt{r}_4\}$ be the epistemic logic programs (ELPs), consisting of the rules:

$$\mathtt{r}_1 = a \,\mathtt{or}\, b. \qquad \mathtt{r}_2 = a \leftarrow \mathsf{K}b. \qquad \mathtt{r}_3 = b \leftarrow \mathsf{K}a. \qquad \mathtt{r}_4 = \bot \leftarrow \mathtt{not}\, \mathsf{K}a.$$

As agreed by the majority, Ψ has a unique world-view $\{\{a\}, \{b\}\}$ due to knowledge-minimisation. Note that $\{\{a\}, \{b\}\}$ fails to satisfy $\mathtt{r}_4$. Thus, with SCM being applied, $\Psi' = \Psi \cup C$ has no world-view. However, each ES_{X} for $x \in \{15, 16, 18\}$, produces the unique world-view/AEEM $\mathscr{A} = \{\{a, b\}\}$ for Ψ'. As

SCM is a special case of ESP, their result contradicts both properties. Moreover, $\mathscr{A}$ also conflicts with FP since $\{\langle\{a\},\mathscr{A}\rangle,\langle\{b\},\mathscr{A}\rangle\}$ is an unfounded set. On the other hand, Cabalar et al. have already proved in separate papers that ES_{20} satisfies all three properties above. Thus, ES_{20} follows their result and yields no AEEMs for $(\Psi')^*$. Thanks to its relational minimality condition (2), ES_{21} does not produce an AEEM for $(\Psi')^*$ either: note that the only candidate $\mathscr{A}$ is not truth-minimal as the weaker per (2) S5-model $\{(\{a\},A),(\{b\},A)\}$ also satisfies $(\Psi')^*$ where $A=\{a,b\}$, so the knowledge-minimality check is redundant. However, if we replace (2) with the functional minimality (1) in ES_{21}, then $\mathscr{A}$ becomes truth-minimal for both $(\Psi')^*$ and $(\Psi)^*$ as none of the weaker per (1) S5-models $\{(\{a\},A)\}$, $\{(\{b\},A)\}$, and $\{(\emptyset,A\}$ satisfies $(\Psi')^*$ or $(\Psi)^*$. As for knowledge-minimality, neither $\mathscr{A}\in\mathtt{EEM}_{21}((\Psi')^*)$ nor $\mathscr{A}\in\mathtt{EEM}_{21}((\Psi)^*)$ has a proper $\mathsf{SW5}$-extension in the same sets, so that makes $\mathscr{A}$ an $\mathtt{AEEM}_{21}$-model for $(\Psi')^*$ and $(\Psi)^*$: note that among all possible proper $\mathsf{SW5}$-extensions $\{\{a,b\},\underline{\{a\}}\}$, $\{\{a,b\},\underline{\{b\}}\}$ and $\{\{a,b\},\underline{\emptyset}\}$ of $\mathscr{A}$, none of them is in $\mathtt{EEM}_{21}((\Psi')^*)$ because they are not $\mathtt{EHT}_{21}$-models of $(\Psi')^*$ or $(\Psi)^*$.

At this point, we need to evaluate formally if such properties (in their original form) may indeed be too restrictive to reveal desired solutions. For a similar informal analysis of Ψ', we refer the reader to [22]. To begin with, we translate Ψ' into the corresponding EHT-formula $(\Psi')^*{=}(a\vee b)\wedge(\mathsf{K}b\to a)\wedge(\mathsf{K}a\to b)\wedge(\neg\neg\mathsf{K}a)$, where the last conjunct $\mathtt{r}_4^*$ is EHT-equivalent to $\mathsf{K}\neg\neg a$, i.e., $\mathsf{K}(\neg a\to\bot)$ by Coroll. 1 in [28]. So, one can interpret $\mathtt{r}_4$ in the way of applying the constraint $\bot{\leftarrow}\mathtt{not}a$ *everywhere*. Note that world-views are S5-models in which any world is designated (actual). Thus, replacing $\mathsf{K}\neg\neg a$ by $\neg\neg a$ in $(\Psi')^*$ normally should not alter the result. If our main priority is to propose a conservative extension of ASP, then $\{\mathtt{r}_1,\mathtt{r}_4\}$ is expected to derive a everywhere since it performs similarly to $\{\mathtt{r}_1,\mathtt{notnot}a\}$ in essence. So, a automatically appears in every world of a possible model. Then, r_3 and r_4 guarantee $\mathscr{A}$ as a world-view of Ψ'. Here, the tricky point is that ES_{20}'s underlying monotonic base EHT_{20} uses KD45-models, and $\mathsf{K}\varphi\to\varphi$ (*the knowledge or truth axiom*) is not a theorem of KD45. Thus, replacing $\mathsf{K}\mathtt{notnot}a$ by $\mathtt{notnot}a$ may result in serious changes in ES_{20} and is not allowed. However, the relational truth-minimality (2) does not allow us to produce $\mathscr{A}$ even for the program $\{\mathtt{r}_1,\mathtt{r}_2,\mathtt{r}_3,\mathtt{notnot}a\}$ either, while functional truth-minimality (1) does. Then, may the condition (2) be eliminating understandable results? To say the least, it is questionable to have no model for $\{\mathtt{r}_1,\mathtt{r}_2,\mathtt{r}_3,\mathtt{notnot}a\}$.

Generally speaking, K represents belief in ES_{20}, whereas it formalises knowledge or being provable in the other semantics of ES. As their major distinguishing feature, we can *believe* a statement to be true when it is false, but it is impossible to *know/prove* a false statement. Thus, $\mathsf{K}p$ has no world-views in ES_{20} as $\{\{p\},\underline{\emptyset}\}$ is a proper KD45-extension of $\{\{p\}\}$. Expectedly, $\{\{p\}\}$ is its unique $\mathtt{AEEM}_{21}$-model. However, this result is understandable as belief on p does not imply its truth. As a result, it may not be a good idea to compare ES_{20} with the other formalisms of ES, including ES_{21}. Instead, we can categorise it separately. As for the suitable epistemic extension of ES, traces from autoepistemic logic also exist in ASP. Remember that $\mathtt{not}p$ reads: p is believed not to hold under the lack of evidence to drive p. Moreover, characterisation of stable models in nonmonotonic KD45 is well-known, and there exists translations between AEL and reflexive AEL, preserving the notion of expansion [16]. However, the latter reflects default reasoning better. In our opinion, $\mathsf{K}p$ is expected to mean in ES: p is derived in *all* worlds. So, interpreting K as *known* may be more appropriate to us, but it should be further discussed.

From a different perspective, we can also argue that $\mathtt{r}_4$ and $\mathtt{r}_1$ are not strong enough to generate $\mathsf{K}a$, which also seems reasonable. Then, we cannot expect to have a world-view. However, we can trigger paracoherent reasoning for ES, as studied in ASP [1] if we really need to obtain an answer for the program. In this case, the literal readings of these rules are: a is *assumed* to hold everywhere in the possible model, and also a or b is *minimally* the case in each world of this model. Thus, the EHT-model[9] $\{\{a\},(\{b\},\{a,b\})\}$, in which the total HT-model $(\{a\},\{a\})$ is simplified into the valuation $\{a\}$, precisely captures the meaning of this statement, further making $\mathtt{r}_3$ and $\mathtt{r}_4$ inapplicable as desired. We leave the use of nontotal EHT-models as a relaxation of world-views to be discussed in future work.

Apart from being reliable tools for ES, first, ESP is not a conservative extension of ASP's standard splitting property (SSP), i.e., a regular ASP-program that can be nontrivially split w.r.t. SSP may not be splittable w.r.t. ESP. Second, FP is designed to weed out unsupported world-views of ES_{91} and cannot guarantee that a founded S5-model of an ELP is also its world-view. Remember that the set of founded classical models of an ASP-program equals the set of its answer sets. What if ES_{91} does not provide a world-view for an ELP, but this result is unintended? Moreover, FP cannot ensure the well-founded classical S5-models w.r.t. knowledge-minimisation. Note that $\{\{a\}\}$ is a founded S5-model of $a\,\mathtt{or}\,b$ w.r.t. FP; yet it is unintended. Briefly, in our opinion, these properties at least need to be strengthened before we regard them as the mandatory criteria that a semantics of ES should comply with.

3.2 Some interesting validities of EHT that are inherited from HT

Now, we extend some well-known propositions of HT to EHT, which we use later for a correct understanding of the real behaviour of complex programs. First, recall that a formula $\varphi \in \mathscr{L}_{\mathsf{EHT}}$ is *satisfiable* if it has an EHT-model. If every EHT-model satisfies φ, then it is *valid* ('$\models_{\mathsf{EHT}} \varphi$'). Given $\varphi,\psi \in \mathscr{L}_{\mathsf{EHT}}$, ψ is a *logical consequence* of φ in EHT ('$\varphi \models_{\mathsf{EHT}} \psi$') if every EHT-model of φ satisfies ψ. When $\varphi \models_{\mathsf{EHT}} \psi$ and $\psi \models_{\mathsf{EHT}} \varphi$ (i.e., they have the same EHT-models), we call them *logically equivalent* in EHT.

Proposition 1 (de Morgan laws and the weak law of the excluded middle both hold in EHT.)

$$\models_{\mathsf{EHT}} \neg(\varphi \wedge \psi) \leftrightarrow \neg\varphi \vee \neg\psi \qquad\qquad \models_{\mathsf{EHT}} \neg\varphi \vee \neg\neg\varphi$$
$$\models_{\mathsf{EHT}} \neg(\varphi \vee \psi) \leftrightarrow \neg\varphi \wedge \neg\psi \qquad\qquad \models_{\mathsf{EHT}} \neg\neg\neg\varphi \leftrightarrow \neg\varphi$$

Proposition 2 For $\varphi,\chi,\psi \in \mathscr{L}_{\mathsf{EHT}}$, the following formulas are logically equivalent in EHT:

i.) $\models_{\mathsf{EHT}} (\neg\neg\varphi \wedge \chi \to \psi) \leftrightarrow (\chi \to \neg\varphi \vee \psi)$ ii.) $\models_{\mathsf{EHT}} (\neg\varphi \wedge \chi \to \psi) \leftrightarrow (\chi \to \neg\neg\varphi \vee \psi)$

Corollary 1 As an immediate consequence of Prop. 2 (hint: take $\chi = \top$), we have: for $\varphi,\psi \in \mathscr{L}_{\mathsf{EHT}}$,

i.) $\models_{\mathsf{EHT}} (\neg\neg\varphi \to \psi) \leftrightarrow (\neg\varphi \vee \psi)$ ii.) $\models_{\mathsf{EHT}} (\neg\varphi \to \psi) \leftrightarrow (\psi \vee \neg\neg\varphi)$

3.3 Comparison between semantics proposals of ES via some critical examples

As mentioned above, AEEMs are in the form of classical S5-models. ES_{15} chooses the AEEMs of a formula φ from the set $\mathtt{EEM}_{15}(\varphi)$ of the candidates. Differently from ES_{20} and ES_{21}, ES_{15} executes a pairwise comparison to the members of this set to guarantee knowledge minimisation: for instance, when $\mathtt{EEM}_{15}(\varphi)=\{\mathscr{A}_1,\mathscr{A}_2\}$, we eliminate $\mathscr{A}_1$ if $\mathscr{A}_1 \subset \mathscr{A}_2$ or $\mathscr{A}_1 <_{\varphi} \mathscr{A}_2$, and so we get $\mathscr{A}_2 \in \mathtt{AEEM}_{15}(\varphi)$. This strategy fails when we add a constraint which $\mathscr{A}_2$ violates because then $\mathscr{A}_1 \in \mathtt{AEEM}_{15}(\varphi)$ rather than having no AEEMs. On the other hand, ES_{20} tests the members of $\mathtt{EEM}_{20}(\varphi)$ according to whether they have a proper KD45-extension in $\mathtt{EEM}_{20}(\varphi)$, and so adding constraints do not cause inconsistencies. More explicitly, $\mathtt{AEEM}_{20}(\varphi)=\{\mathscr{A}_2\}$ when $\mathtt{EEM}_{20}(\varphi)=\{\mathscr{A}_1,\mathscr{A}_2,\mathscr{A}_3\}$ where $\mathscr{A}_3$ is a proper KD45-extension of $\mathscr{A}_1$. However, adding a subjective constraint which is not satisfied by $\mathscr{A}_2$ causes the lack of AEEMs for φ. The case for arbitrary constraints should further be checked. Note that ES_{16} and ES_{18} also suffer from a similar pairwise comparison of possible candidates. The following example illustrates this discussion.

[9] When we use no subscript such as EHT_{20}, EHT is accepted to be EHT_{15} by default, i.e., the combination of S5 and HT.

Example 2 (given by Cabalar et al. [4] to show that ES_{15}, ES_{16}, and ES_{18} violate epistemic splitting) Let $\Sigma = \{\mathtt{r}_1, \mathtt{r}_2, \mathtt{r}_3\}$ be the epistemic logic program (ELP), consisting of the rules given below:

$$\mathtt{r}_1 = a \,\mathtt{or}\, b. \qquad \mathtt{r}_2 = c \leftarrow \mathsf{K}a. \qquad \mathtt{r}_3 = \bot \leftarrow \mathtt{not}c. \quad (\text{ or, } \mathtt{r}_3' = \mathtt{not}\mathtt{not}c.)$$

First take $\Sigma_1 = \{\mathtt{r}_1, \mathtt{r}_2\}$: it has a unique and clearly understandable world-view $\mathscr{A}_1 = \{\{a\}, \{b\}\}$ in ES_{16} and ES_{18}. Note that $\mathscr{A}_2 = \{\{a,c\}\}$ does not occur as a truth-minimal model of Σ_1 in ES_{16} and ES_{18}, thanks to their fixed point equations $\overset{\mathsf{fp}}{=}$. However, in ES_{15} and ES_{20}, we have both $\mathscr{A}_1$ and $\mathscr{A}_2$ as truth-minimal EEMs respectively according to the tools 1 and 2. Fortunately, they eliminate $\mathscr{A}_2$ w.r.t. their knowledge-minimisation properties. Then, consider the whole program Σ: now, ES_{15}, ES_{16} and ES_{18} all withdraw $\mathscr{A}_1$ since it violates the constraint $\mathtt{r}_3$ and instead choose $\mathscr{A}_2$ as the unique world-view/AEEM: for Σ and $\mathscr{A}_2$, the fixed point equations of ES_{16} and ES_{18} hold, and now there is no rival. To us, this result provided by $\mathscr{A}_2$ is unsupported: while an agent disjunctively has two alternative information, a and b, about a world, she cannot justify $\mathsf{K}a$. So, $\mathtt{r}_2$ becomes inapplicable and the existence of c is unfounded. Further inserting the constraint $\mathtt{r}_3$ can guarantee neither $\mathsf{K}a$ nor c. Thus, Σ should have no world-views/AEEMs as is the case in ES_{20} because $\{\{a,c\}\} \in \mathtt{EEM}_{20}(\Sigma)$ has the proper $\mathsf{KD45}$-extension $\{\{a,c\}, \underline{\{b,c\}}\}$ in $\mathtt{EEM}_{20}(\Sigma)$. As expected, Cabalar et al.'s principle of ESP aligns with the result of ES_{20}. As $\mathtt{AEEM}_{21}(\Sigma) = \{\mathscr{A}_2\}$, we show by this counterexample that ESP does not hold for ES_{21}. Note that $\mathscr{A}_2 \in \mathtt{EEM}_{21}(\Sigma)$ has no proper $\mathsf{SW5}$-extension in the same set: the only candidate does not hold as $\{\{a,c\}, \underline{(\{b\}, \{b,c\})}\}$ satisfies Σ.

Example 3 (used by Kahl as a motivating example for his new modal reduct first given in [12]) Take the ELP $\Delta = \{\mathtt{r}_1, \mathtt{r}_2\}$ where $\mathtt{r}_1 = a \,\mathtt{or}\, b$ and $\mathtt{r}_2 = b \leftarrow \mathsf{M}\,a$, and then translate it into the corresponding EHT-formula $\Delta^* = (a \vee b) \wedge (\neg\mathsf{K}\neg a \to b)$. We know that $\neg\neg\hat{\mathsf{K}}$, $\hat{\mathsf{K}}\neg\neg$ and $\neg\mathsf{K}\neg$ are all equivalent in EHT (see Prop. 5; [28]) So, using Coroll. 1, we deduce that Δ^* is equivalent to $(a \vee b) \wedge (b \vee \neg\hat{\mathsf{K}}a)$ in EHT, and again by Prop. 5 [28], even further to $b \vee (a \wedge \mathsf{K}\neg a)$. Note that the last disjunct yields a contradiction in EHT_{15} and EHT_{21}, making Δ^* and b EHT-equivalent. Thus, Δ has the unique AEEM $\{\{b\}\}$ in ES_{15} and ES_{21}; yet ES_{20} gives no AEEMs as $\{\{b\}\}$ has a proper $\mathsf{KD45}$-extension $\{\{b\}, \underline{\{a\}}\}$ in $\mathtt{EEM}_{20}(\Delta)$. Δ cannot be split w.r.t. ESP. However, $\{\{b\}\}$ is a founded model of Δ w.r.t. FP. So, a semantics satisfying FP is supposed to have this world-view. Semantics like ES_{20} and ES_{91} jump over this test since they do not have world-views for Δ. This is why we find it essential to reinforce FP so as to guarantee that a successful semantics should be able to bring out all founded S5-models of an ELP as its world-views/AEEMs.

Example 4 (discussed by Cabalar et al. [4] to show that ES_{15}, ES_{16}, and ES_{18} violate epistemic splitting) Let $\Upsilon = \{\mathtt{r}_1, \mathtt{r}_2, \mathtt{r}_3, \mathtt{r}_4\}$ be the epistemic logic program (ELP), composed of the following rules:

$$\mathtt{r}_1 = a \,\mathtt{or}\, b. \qquad \mathtt{r}_2 = c \,\mathtt{or}\, d \leftarrow \mathtt{not}\mathsf{K}a. \qquad \mathtt{r}_3 = \bot \leftarrow c. \qquad \mathtt{r}_4 = \bot \leftarrow d.$$

Then, $\Upsilon^* = (a \vee b) \wedge (\neg\mathsf{K}a \to c \vee d) \wedge (\neg c) \wedge (\neg d)$. By Prop. 1 and Coroll. 1, Υ^* is equivalent to $(a \vee b) \wedge ((c \vee d) \vee \neg\neg\mathsf{K}a) \wedge \neg(c \vee d)$. Using Coroll. 1 in [28], we further simplify Υ^* into $(a \vee b) \wedge (\mathsf{K}\neg\neg a) \wedge \neg(c \vee d)$. Thus, this formula, in essence, has the same meaning as $(a \vee b) \wedge (\neg\neg a) \wedge \neg c \wedge \neg d$, whose unique world-view/AEEM is $\{\{a\}\}$ in each ES_{x} for $x \in \{15, 16, 18, 20, 21\}$ w.r.t. supra-ASP. So, for a semantics of ES with classical S5-models (i.e., according to supra-S5), $\{\{a\}\}$ is expected to be the only world-view/AEEM for Υ. Nonetheless, ES_{20} has no AEEMs for Υ because the unique possibility $\{\{a\}\}$ has the proper $\mathsf{KD45}$-extension $\{\{a\}, \underline{\{b\}}\} \in \mathtt{EEM}_{20}(\Upsilon)$. Of course, this result is normal because reflexivity is not valid in EHT_{20}, and so it is not legal to make such transitions in it. However, we can assert that the knowledge-minimisation technique of AEL may not be the best choice to be employed in ES. Note that ES_{21}, using the reasoning of RAEL, obtains the AEEM $\{\{a\}\}$ for Υ^* as $\{\{a\}, \underline{\{b\}}\} \not\models_{\mathsf{EHT}_{21}} \Upsilon^*$. As an advantage, extending ES_{21} with world-view constructs [13] will then make ES_{21} more expressive than ES_{20}. Also, SCM is useful in problem descriptions of some domains like conformant planning [13, 4].

4 Conclusion

The main purpose of this paper is to carefully revise the competing approaches of ES, among which are ES_x for $x \in \{15, 16, 18, 20\}$. We systematically bring to light the (dis)advantages of these formalisms. In doing so, we discuss how we can reach a more suitable epistemic extension of ASP. We also propose a slightly new formalism called ES_{21}, which can also be regarded as reflexive ES_{20}. We do so because ES_{20} uses a well-studied technique of knowledge minimisation, but it is a nonmonotonic epistemic logic of belief, while all the rest can be considered as epistemic formalisms of knowledge. As future work, we will first establish a strong equivalence characterisation of ELPs under the ES_{21}-semantics, which is identified as EHT_{21}-equivalence. Then, we also would like to study paracoherent semantics of ELPs.

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