The Forms of Categorical Proposition

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An exhaustive survey of categorical propositions is proposed in the present paper, both with respect to their nature and the logical problems raised by them. Through a comparative analysis of Term Logic and First-Order Logic, it is shown that the famous problem of existential import may be solved in two ways: with a model-adaptive strategy, in which the square of opposition is validated by restricting the models; with a language-adaptive strategy, in which the logical form of categorical propositions is extended in order to validate the square in every model. The latter strategy is advocated in the name of logic, which means truth in every model. Finally, the present paper needs some automatic process in order to determine the nature of logical relations between any pair of the available 256 categorical propositions. This requires the implementation of a programming machine in the style of Prolog.

1 What is a categorical proposition?

It is taken for granted that the roots of modern logic stem from Aristotle's syllogistics, as a general inferential activity. However, the content of such inferences was initially restricted to a specific kind of formulas: *categorical propositions*. A general definition of the latter is the following: by a categorical proposition, it is meant any proposition in which something, say *A*, is affirmed or denied of something, say *B* (*A* and *B* may be two names of the same thing).[1] Every thought may be expressed in that way, provided that the grammatical form of the proposition includes a first term, the subject, and a second term, the predicate.

The aim of the following is not to deal with syllogistics as it stands; rather, we want to deal with the various ways in which categorical propositions may be analyzed, especially in the light of their logical form. A number of questions may arise from this perspective: 'Every *A* is *B*' is a typical categorical proposition, whereas 'Everything is *A* and *B*' doesn't seem so. Isn't it? Does 'Every *A* is *B*' entail that 'Some A is B'? What is the difference between '*A* is not-*B*' with respect to '*A* is not *B*'? Here are some issues that are tackled in the following work, in order to give a comprehensive survey of what is meant by the so-called 'categorical propositions'.

A basic proposition is of the form '*A* is *B*' and constitutes the core expression of the so-called traditional Term logic (henceforth: TL)(see [7]). According to TL, every proposition is a pair of terms bound together by a (logical) copula to form a single unified expression. Modern logic results from a choice: for a number of reasons, Frege defeated Aristotle and the contemporary First-Order Logic (henceforth: FOL) is the mainstream formal language that is mainly used in mathematics and the other formal sciences. Assuming that the value of a language lies in its expressive power, FOL is expected to be able to express as much of our thoughts as Term Logic. This is not so easy in any case, however. Let us consider

[1] Although, literally speaking, a categorical proposition also includes an expression of *quantity* (in order to say how many elements of the *A* are *B*), the above definition may be viewed as a categorical *proto-proposition* (see [3]), i.e. the core expression from which any categorical proposition and its extensions may be formed. We are grateful to one of the anonymous reviewers for that relevant note about what categoricity traditionally means.

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(1) Ryan is even.

(1) is expected to be false, for 'even' is to be predicated of whatever is a number only. So the propositional negation of (1),

(1)* Ryan is not even,

is true by *denying* a predicate that does not belong to the properties of whatever is not a number. But, as we will see in the next section, there are other negative expressions that are not applied to entire propositions and occur inside the propositions, e.g. the affixal negations like 'undefeated' or 'impossible'. Let us call *term negations* those cases in which negation is applied only to one term of the proposition (e.g. 'defeated' and 'possible', in the previous two examples), to be symbolized by a hyphen in order to make the difference between propositional negation in '*A* is not *B*' and '*A* is not-*B*.[2] Traditional logic made a distinction between propositional negation and term negation, accordingly: (1)* means something like 'It is not the case that Ryan is even', whereas

(2) 9 is not-even

is true by *affirming* that the number 9 is not-even, that is, odd.
FOL is not able to express term negation, because the traditional copula finds no expression in it and this reduces negation to a unique propositional operator. Whilst (1)* is true, the other proposition

(1)′ Ryan is not-even

is false since being not-even is synonymous with being odd. TL makes this distinction in terms of distinctive *oppositions* (see [4], [8]): '*A* is *B*' and '*A* is not *B*' are said to be contradictories because the truth of the one entails the falsehood of the other and conversely; at the same time, '*A* is *B*' and '*A* is not-*B*' are said to be *contraries* because the truth of the one entails the falsehood of the other whilst both can be false (Ryan is neither even nor odd, indeed).
Although FOL lacks expressive power in this respect, it may neglect this point by recalling that formal logic essentially deals with generality and is not interested with individual truths (see [5]). Aristotle's logic confirms this point by giving priority to general propositions about classes of individuals, like the *A*s or the *B*s. As FOL is able to rephrase every such proposition, the previous semantic distinction between (1)* and (1)′ is ignored. Nonetheless, let us now consider

(3) Dragons are dangerous.

(3) is admittedly said true in most of its fictional statements. But the corresponding proposition is supposed to be false for whomever has a robust sense of reality, because dragons do not exist. So the contradictory of (3) is true, i.e. (3)* Dragons are not dangerous, by definition. Then what of its presumed contrary, i.e.

(3)′ Dragons are not-dangerous,

[2] This is confirmed by Aristotle in his *Prior Analytics*, I,46: "It is clear that 'is not-white' and 'is not white' signify different things and that one is an affirmation, the other a denial." (52a24-26). (Also quoted in Read 2015: 9).

which affirms something and means that dragons are not-dangerous, that is, friendly? $(3)'$ is as false as (3), if dragons do not exist. This is a way to say that the Aristotelian square of opposition includes some ambiguity behind its usual headings. The proposition ‘Every A is B’ is called *affirmative universal*, because being B is affirmed of every A, whereas the proposition ‘Every A is not B’ is called *negative universal* because being B is denied of every A. Then ‘Every A is not-B’ should be considered as a special case of affirmative universal, in which the negative predicate not-B is affirmed of every A. The point is that there is no way to express the difference between ‘Every A is not B’ and ‘Every A is not-B’ in FOL, or it seems so.

The following purports to present both a syntax and a semantics for all these kinds categorical propositions without exception. In Section 2, a variety of logical forms is listed from proto-categorical to apodictic propositions. In Section 3, their truth-conditions are expressed in terms of ordered model sets.

2 Syntax

A clear-cut difference is to be made between two kinds of speech act, namely: affirming a negative predicate term A, ‘is not-A’, and denying a positive predicate A, ‘is not A’. The latter corresponds to what Aristotle called the *composition* of a proposition in its ultimate stage. That is: asserting A of something means that the resulting proposition is taken to be *true*, whereas denying A means that the resulting proposition is taken to be *false*. The truth-conditions of propositions will be studied in the next section. Here is a set of increasingly complex logical forms of propositions, $\mathscr{A}_C^n$, where n stands for the number of *literals*, i.e. components of a compound formula that can either affirmed (+) or denied (–). The logical complexity of propositions depends on the number of literals that are in it.

Type 1: ‘A is (not) B’.
$\mathscr{A}_C^1 = A \pm \text{is } B$
$2^1 = 2$ kinds of proposition: (1) A is B, (2) A is not B.

Type 2. ‘all/some A is (not) B’.
$\mathscr{A}_C^2 = \pm\text{some } A \pm \text{is } B$.
$2^2 = 4$ kinds of proposition: (1) some A is B, ..., (4) all A is not B.

Type 3. ‘(not-)A is (not) (not-)B’.
$\mathscr{A}_C^3 = \pm A \pm \text{is } \pm B$
$2^3 = 8$ kinds of proposition: (1) A is B, ..., (8) not-A is not not-B.

Type 4. ‘(all/some) (not-)A is (not) (not-)B’.
$\mathscr{A}_C^4 = \pm\text{some } \pm A \pm \text{is } \pm B$
$2^4 = 16$ kinds of proposition: (1) some A is B, ..., (16) all not-A is not not-B.

Type 5. (all/some) (not-)A is (not) (necessarily/possibly) (not-)B
$\mathscr{A}_C^5 = \pm\text{some } \pm A \pm \text{is } \pm\text{possibly } \pm B$
$2^5 = 32$ kinds of proposition: (1) some A is possibly B, ..., (32) all not-A is necessarily not-B.

The rest of the paper will be confined to dealing with assertive propositions, i.e. the set of categorical propositions including Aristotle's four copulae. Given that this class of proposition includes $n = 4$ literals in the syntax of TL, we will symbolize henceforth any categorical proposition by A^4_C accordingly.

In light of FOL, we take for the time being the following propositional scheme to be the common logical form of any categorical propositions:

$$\mathscr{A}^4_C = \pm\exists x(\pm Ax \wedge \pm Bx)$$

The 3 literals that can be either affirmed or denied, viz. existential quantifier and the two predicates, lead to a set of $2^3 = 8$ distinct formulas, i.e. 4 categorical propositions X with only affirmed subject terms and 4 categorical propositions X' with only denied subject terms.

Given that the propositional negation of the latter is logically equivalent to a universal, the following table affords a set of two possible formalization for categorical propositions: either in FOL, or in TL.

	FOL	TL
A	$\neg(\exists x)(Ax \wedge \neg Bx)$	$-(+A-(+B))$
E	$\neg(\exists x)(Ax \wedge Bx)$	$-(+A+B))$
I	$(\exists x)(Ax \wedge Bx)$	$+(+A+B)$
O	$(\exists x)(Ax \wedge \neg Bx)$	$+(+A+(-B))$
A′	$\neg(\exists x)(\neg Ax \wedge Bx)$	$-(+A+B)$
E′	$\neg(\exists x)(\neg Ax \wedge \neg Bx)$	$-(-A+(-B))$
I′	$(\exists x)(\neg Ax \wedge \neg Bx)$	$+(-A+(-B))$
O′	$(\exists x)(\neg Ax \wedge Bx)$	$+(-A+B)$

Although Aristotle's logical works were usually devoted to scientific discourse and arguably led to some disregard for singular propositions, we have seen that the latter may be analyzed on their own for purely logical reasons. In the same vein, there is a semantic reason why the Aristotelian theory of categorical propositions has usually been restricted to the set $\{\mathbf{A},\mathbf{E},\mathbf{I},\mathbf{O}\}$, whereas the other formulas $\{\mathbf{A}',\mathbf{E}',\mathbf{I}',\mathbf{O}'\}$ were always left out of consideration.
So let us consider this semantics, before turning to a general consideration of categorical propositions and their proper logical form.

3 Semantics

Our main concern will be with models. *Logical truth* means that a proposition is supposed to be true in every model, whilst a *logical consequence* means that models of one formula are also models of entailed formulas. Although these general features of logic are assumed to hold in every model, some convenient restrictions are generally made in order to warrant the completeness of logical systems. One of these is the non-emptiness of models: What is the truth of 'Every x is x' in an empty model, i.e. the model in which there is nothing? Whilst the corresponding formula $(x)(x = x)$ refers to the law of self-identity, it is equally true to affirm 'No x is x' in the empty model, so that the contrary formula $\neg(\exists x)(x = x)$ is also true in it and, by classical logic, $(x)(x \neq x)$. Incompleteness is on a par with inconsistency, accordingly: classical logic cannot cope with emptiness. The same difficulty arises in modal logic, insofar as necessary and impossible truths are logically equivalent as well in the empty model. Does it follow from it that any logical system should always prohibit emptiness and make some minimal existential assumption for its own sake? Let us consider what happens in the contrary case, with the special case of categorical propositions and traditional logic.

3.1 Existential import

A semantics for categorical propositions is expected to afford the truth-conditions relative to a set of models these propositions belong to. Let us call 'model set' all those models where a given proposition is true. A model is a set of propositions including, e.g., φ, that are true in wi if, and only if (henceforth: iff), they belong to w_i: $v(w_i, \varphi) = 1$ iff $\varphi \in w_i$; and they are false in w_i, otherwise. We write $v(w_i, \varphi) = 1$ for 'φ is true in w_i', and $v(w_i, \varphi) = 0$ for 'φ is false in w_i'. As categorical propositions $\mathscr{A}_C^n$ are used to be analyzed into a bivalent frame, their corresponding models are expected to be consistent and complete. That is: propositions are either true or false in any model: $v(w_i, \mathscr{A}_C^n) = 1$ iff $v(w_i, \neg\mathscr{A}_C^n) = 0$.
One theory that famously deals with the logical relations between categorical propositions is the *theory of opposition* (henceforth: *TO*). According to *TO*, a set of logical relations holds between any propositions in every model: contrariety, contradiction, subcontrariety, and subalternation. Each of these logical relations can be defined in terms of constraints upon the truth-values of their relata. Thus, for any propositions A, B and any model w_i:

A and B are contraries iff $v(w_i,A) = 1$ entails $v(w_i,B) = 0$.
A and B are *contradictories* iff $v(w_i,A) = 1$ entails $v(w_i,B) = 0$ and $v(w_i,A) = 0$ entails $v(w_i,B) = 1$.
A and B are *subcontraries* iff $v(w_i,A) = 0$ entails $v(w_i,B) = 1$.
B is the *subaltern* of A iff $v(w_i,A) = 1$ entails $v(w_i,B) = 1$ and $v(w_i,B) = 0$ entails $v(w_i,A) = 0$.

More generally, [9] rephrased all these logical relations into two main sets of a basic relation of entailment (symbol: $\vdash$): compatibility, and incompatibility. Opposition is initially a synonym for incompatibility, while subcontrariety and subalternation are further logical relations between compatible propositions. It concerns contrariety and contradiction and means that A and B are incompatible (symbol: $\perp$) iff these cannot be both true in the same model:

$$A \perp B =_{df} v(w_i,A) = 0 \text{ iff } v(w_i,B) = 1.$$

Or, equivalently, A and B are incompatible with each other iff the truth of A in w_i entails the falsity of B, i.e. the truth of $\neg B$:

$$A \perp B =_{df} A \vdash \neg B.$$

Although *TO* was initially devoted to the logical relations between categorical propositions through Aristotle's logical treatises, it has then be extended to any kind of propositional expressions. This theory consists in claiming that the aforementioned propositions stand in the same logical relations in any model whatsoever. Thus, *TO* states that, for every w_i, the following holds in the set of Aristotelian categorical propositions, $\mathbf{X}$:

$\mathbf{A} \vdash \neg\mathbf{E}, \mathbf{A} \vdash \neg\mathbf{0}, \neg\mathbf{A} \vdash \mathbf{O}, \mathbf{A} \vdash \mathbf{I}$,
$\mathbf{E} \vdash \neg\mathbf{A}, \mathbf{E} \vdash \neg\mathbf{I}, \neg\mathbf{E} \vdash \mathbf{I}, \mathbf{E} \vdash \mathbf{O}$,
$\mathbf{I} \vdash \neg\mathbf{E}, \neg\mathbf{I} \vdash \neg\mathbf{A}, \neg\mathbf{I} \vdash \mathbf{E}, \neg\mathbf{I} \vdash \mathbf{O}$,
$\mathbf{O} \vdash \neg\mathbf{A}, \neg\mathbf{O} \vdash \mathbf{I}$.

Exactly the same set of logical relations hold for their complementaries, i.e. the Keynesian categorical propositions, $\mathbf{X}'$:

$\mathbf{A}' \vdash \neg\mathbf{E}', \mathbf{A}' \vdash \neg\mathbf{O}', \neg\mathbf{A}' \vdash \mathbf{O}', \mathbf{A}' \vdash \mathbf{I}'$,

$\mathbf{E}' \vdash \neg\mathbf{A}', \mathbf{E}' \vdash \neg\mathbf{I}', \neg\mathbf{E}' \vdash \mathbf{I}', \mathbf{E}' \vdash \mathbf{O}'$,
$\mathbf{I}' \vdash \neg\mathbf{E}', \neg\mathbf{I}' \vdash \neg\mathbf{A}', \neg\mathbf{I}' \vdash \mathbf{E}', \neg\mathbf{I}' \vdash \mathbf{O}'$,
$\mathbf{O}' \vdash \neg\mathbf{A}', \neg\mathbf{O}' \vdash \mathbf{I}'$.

Now [13] recalled that *TO* does not hold in every model but only in those models where the subject term A is not 'empty'. An empty term being a predicate that applies to no individual, this means that *TO* holds only in those models in which something instantiates A, i.e. every wi such that $v(w_i, Ax) = 1$. For let us suppose the contrary, i.e. a model w_j in which nothing is A. Then $v(w_j, \exists xAx) = 0$ and, hence, $v(w_j, \mathbf{A}) = v(w_j, \mathbf{E}) = 1$, and $v(w_j, \mathbf{I}) = v(w_j, \mathbf{O}) = 0$. This means that none of the logical relations of *TO* holds in wj. And given that *TO* is expected to hold in every model, *TO* fails.
A number of solutions have been proposed to revalidate *TO*, and the related literature is famously referred to since medieval logic as the problem of existential import. According to their authors, some categorical propositions 'have' import by naturally entailing the existence of their subject term: $(w_i, \mathscr{A}^n_C \vdash \exists xAx)$ for any w_i; whereas the other ones do not, or 'lack' import. Two main options are proposed in this respect: import by quality, (I_1), and import by quantity, (I_2). According to (I_1), the 'affirmative' propositions **A** and **I** always have import whereas **E** and **O** lack import. And according to (I_2), the 'existential' propositions **I** and **O** always have import (which accounts for how the traditional particular quantifier became the existential quantifier) whereas **A** and **E** lack import.
Another problem raised by existential import is about logic as it stands: How can a theory be called 'logical' if it does not hold in *every* model whatsoever? Note finally that the problem with (affirmative) existential import may also be extended in the form of a problem with 'negative existential import': although the literature about opposition has been entirely devoted to the logical relations between the set of Aristotelian propositions $\{\mathbf{A},\mathbf{E},\mathbf{I},\mathbf{O}\}$, *TO* is also invalidated by the set of 'complementary' or Keynesian propositions $\{\mathbf{A}',\mathbf{E}',\mathbf{I}',\mathbf{O}'\}$. For let us suppose now that there is a model w_k in which *everything* is A, so that the subject term A may be said 'full' in w_k. Then the same logical trouble ensues in w_k, for $v(w_k, \mathbf{A}') = v(w_k, \mathbf{E}') = 1$ and $v(w_k, \mathbf{I}') = v(w_k, \mathbf{O}') = 0$. Hence *TO* equally fails with affirmative and negative existential import, and the trouble goes beyond the sole case of 'empty' terms. It also concerns the predicate terms B, whose occurrence was even more neglected than the case in which everything is A; indeed, we will see later that the logical relations between Aristotelian and Keynesian categorical propositions also fails in models in which either nothing is B or everything is B.

Faced with all these difficulties, in the following we will propose a formal solution to the problem of existential import. That is: this problem does not stem from the informal reading of categorical propositions; rather, it relies on the way in which the latter are to be understood in light of their given logical forms. In other words, we want to restore the validity of *TO* by showing that its alleged invalidity is due to an incorrect view about the logical form of A^4_C and does not require any restriction on the models. For the corresponding theory would not be a logical theory properly speaking, otherwise.

3.2 Two Ways of Validity

There are two ways of constructing a semantics for *TO*, in order to obtain a correspondence between a formal language and its model set. Either by restricting the available models in order to maintain the validity of some formulas, in accordance to their logical form. Or by altering these logical forms without imposing any restriction on their corresponding models. Let us call 'model-adaptive strategy' the first semantic method by virtue of which models are restricted in order to validate some logical forms. In contrast, let us call 'language-adaptive strategy' the second semantic method by virtue of which logical forms are adapted in order to be valid in unrestricted models. Although the latter view is the only one that seems to yield a properly 'logical' theory and that will be endorsed in the rest of the paper accordingly, let us consider how to construct a model-adaptive semantics in the next section, in order to validate *TO* in light of the historical tradition. The later section will be devoted to the language-adaptive method, where the solution relies on alternative logical forms for categorical propositions.

3.2.1 Semantics for restricted models (model-adaptive strategy)

The square of opposition is usually or 'normally' said to be valid in non-empty models, i.e. models in which what is predicated exists. Let us call 'normal' these restricted models that make *TO* valid, accordingly. We make use of a *bitstring semantics* for this purpose, in which all logical relations between arbitrary propositions is determined by a complete set of exclusive and exhaustive models in which they hold or not. These complementary models include submodels, and a model-adaptive semantics consists in constructing a set of such models by means of formulas whose logical relations are expected to hold. In the case of Aristotelian categorical propositions, it has been said previously (on page 7) that a number of logical dependencies are supposed to hold between the closed sets of Aristotelian and Keynesian categorical propositions. These logical relations appear in the following table where, for any formulas A, B:

$A \vdash B$ means that every model of A is a model of B.
$A \dashv B$ means that every model of B is a model of A.
$A \dashv\vdash B$ means that every model of A is a model of B and every model of B is a model of A.
$A \perp B$ means that every model of A is not a model of B and every model of B is not a model of A.

	A	**E**	**I**	**O**
A	$\dashv\vdash$	$\perp$	$\vdash$	$\perp$
E	$\perp$	$\dashv\vdash$	$\perp$	$\vdash$
I	$\dashv$	$\perp$	$\dashv\vdash$	
O	$\perp$	$\dashv$		$\dashv\vdash$

The above two empty boxes correspond to the relation of subcontrariety between **I** and **O**: there is a logical dependence between the latter, insofar as the falsehood of either one entails the truth of the other; however, this dependence is a case of compatibility in which not every model of a proposition puts any special constraint by excluding the other. For this reason, **I** and **O** may belong or not to one and the same model and their conjunction **I** $\wedge$ **O** holds there.

The model-adaptive method *TO* relies on the following tenets: the construction of a model consists in finding a set of formulas that are complementary with each other, i.e. incompatible with each other and not entailed by each other; a complete model for a given set of formulas is established once each such formula either entails or is incompatible with the other ones; any empty box of the table means that the corresponding formulas are compatible with each other and their conjunction holds. Each complete set of formulas is called a *bit*, and the ordered combination of bits yields a *bitstring* that characterizes a complete model for a given set of formulas.

In the above table of logical relations, only **A** and **E** either entail or are incompatible with any other formula of the set of Aristotelian categorical propositions, **X**. Hence **A** and **E** correspond to two bits, i.e. two formulas for which distinct models are required. **I** and **O** are not bits, but their conjunction **I** $\wedge$ **O** might be so. Let us construct a further table to check the resulting logical relations.

	A	**E**	**I ∧ O**
A	$\dashv\vdash$	$\perp$	$\perp$
E	$\perp$	$\dashv\vdash$	$\perp$
I ∧ O	$\perp$	$\perp$	$\dashv\vdash$

All the above formulas are incompatible with each other, so the table is closed. It results in a set of 3 bits characterizing the meaning of any formula of A, i.e. the set of its truth-possibilities in terms of models. Thus, for any **X** in a 'normal' model set W^N that validates **TO**, its characteristic bitstring

$$v(W^N, \mathbf{X}) = \langle v(w_1^N, \mathbf{X}), v(w_2^N, \mathbf{X}), v(w_3^N, \mathbf{X})\rangle$$

and corresponds to an ordered set of 3 complementary sets that exhaust the set of truth-possibilities in accordance to *TO*: w_1^N is the set in which **A** holds; w_2^N is the set in which $\mathbf{I} \wedge \mathbf{O}$ holds; and w_3^N is the set in which **E** holds. An arbitrary formula $\mathscr{A}$ holds in a model w_i iff $v(w_i, A) = 1$, and it does not iff $v(w_i, A) = 0$. Here is a table that lists the meaning of Aristotelian categorical propositions in accordance to *TO*.

X	$v(W^N, \mathscr{A}^4_{C_A})$
A	100
E	001
I	110
O	011

The ordered bits that symbolize the meaning of formulas helps to show their logical relations: every model of **A** is also a model of **I**, insofar as every ordered 1-bit of the former is also a 1-bit of the latter. This helps to rephrase logical relations between any formulas A, B in those Boolean terms:

$A \perp B$ means that every ordered 1-bit of A is a 0-bit of B.

$A \vdash B$ means that every ordered 1-bit for A is a 1-bit for B.

The same semantics obtains with the set of Keynesian categorical propositions $\mathbf{X}'$, given that the set of logical relations between its corresponding formulas is identical to those between the Aristotelian categorical propositions. This leads to the isomorphic bitstring representation

$$v(W^N, \mathbf{X}') = \langle v(w_1^N, \mathbf{X}'), v(w_2^N, \mathbf{X}'), v(w_3^N, \mathbf{X}')\rangle$$

where w_1^N is the model set in which **A**' holds; w_2^N is the model set in which **I**' ∧ **O**' holds; and w_3^N is the model set in which $\mathbf{E}'$ holds. Whilst the same bitstrings obtain with $\mathbf{X}'$,

$\mathbf{X}'$	$v(W^N, \mathscr{A}^4_{C_A})$
$\mathbf{A}'$	100
$\mathbf{E}'$	001
$\mathbf{I}'$	110
$\mathbf{O}'$	011

the situation is different with the common set of *interrelations* between Aristotelian and Keynesian categorical propositions, whose corresponding model includes their 4 + 4 = 8 formulas.

	A	**E**	**I**	**O**	**A′**	**E′**	**I′**	**O′**
A	$\dashv\vdash$	$\perp$	$\vdash$	$\perp$		$\perp$	$\vdash$	
E	$\perp$	$\dashv\vdash$	$\perp$	$\dashv$	$\perp$			$\vdash$
I	$\dashv$	$\perp$	$\dashv\vdash$	$\perp$	$\dashv$			
O	$\perp$	$\dashv$		$\dashv\vdash$		$\dashv$		
A′		$\perp$	$\vdash$		$\dashv\vdash$	$\perp$	$\vdash$	$\perp$
E′	$\perp$			$\vdash$	$\perp$	$\dashv\vdash$	$\perp$	$\vdash$
I′	$\dashv$				$\dashv$	$\perp$	$\dashv\vdash$	
O′		$\dashv$			$\perp$	$\dashv$		$\dashv\vdash$

Again, empty boxes mean that their corresponding relata A, B may occur in the same model consistently; these relata are replaced in a new whole table by their conjunction $A \wedge B$ in a new table of whole logical relations, until a final table that includes only incompatible sets of propositions. The process leads hereby to a final set of 7 incompatible 'normal' models for general categorical propositions $\mathscr{A}^4_C = \{\mathbf{X}, \mathbf{X}'\}$, instead of the previous 3 ones for strictly Aristotelian and strictly Keynesian categorical propositions:

$$v(W^N, \mathscr{A}^4_C) = \langle v(w^N_1, \mathbf{X}'), ..., v(w^N_7, \mathbf{X}') \rangle$$

where w^N_1 is the model set in which $\mathbf{A} \wedge \mathbf{A}'$ holds; w^N_2 is the model set in which $\mathbf{A} \wedge \mathbf{O}'$ holds; w^N_3 is the model set in which $\mathbf{A}' \wedge \mathbf{O}$ holds; w^N_4 is the model set in which $\mathbf{I} \wedge \mathbf{O} \wedge \mathbf{I} \wedge \mathbf{O}'$ holds; w^N_5 is the model set in which $\mathbf{I} \wedge \mathbf{E}'$ holds; w^N_6 is the model set in which $\mathbf{E} \wedge \mathbf{I}'$ holds; and w^N_7 is the model set in which $\mathbf{E} \wedge \mathbf{E}'$ holds.

The set of ensuing logical relations between Aristotelian and Keynesian categorical propositions can be gathered from their following corresponding bitstrings:

X	$v(W^N, \mathscr{A}^4_C)$
A	1100000
E	0000011
I	1111100
O	0011111
A′	1010000
E′	0000101
I′	1111010
O′	0101111

where the relation of logical *independence* is rendered in Boolean terms as follows: any two formulas A, B are logically independent from each other iff not every model for A is a model for either B or $\neg A$, that is, i.e. not every ordered 1-bit for A corresponds to either a 1-bit or a 0-bit for B.

3.2.2 Semantics for unrestricted models (language-adaptive strategy)

A second way to validate all the expected logical relations between categorical propositions is by altering their logical forms, instead of restricting models. Given that the central difficulty for *TO* traditionally turns around existential import, the characteristic normal form of $\mathscr{A}^4_C$ should take this into account by introducing two additional clauses: that something is A; and that something is B. Nonetheless, it has been shown that other logical difficulties arise with what we called a 'universal import', i.e. whenever everything is either A or B in a model. Accordingly, the most comprehensive logical form representing all kinds of categorical propositions is

$$\pm\exists x \pm Ax \wedge \pm\exists x \pm Bx \wedge \pm\exists x(\pm Ax \wedge \pm Bx)$$

The above extended logical form includes 7 literals, hence a resulting set of $2^7 = 128$ categorical propositions. These correspond to a set of 8×16 propositions, each of the initial 8 Aristotelian and Keynesian categorical propositions that are expressed by the third conjunct $\pm\exists x(\pm Ax \wedge \pm Bx)$ (and its 3 literals $\pm\exists x$, $\pm Ax$, $\pm Bx$) being now partitioned into $n = 16$ expressions of ontological commitment (about A, $\pm\exists x \pm Ax$, or about B, $\pm\exists x \pm Bx$) that are expressed by the first two conjuncts (and their 4 literals).

Is this new logical form in position to revalidate all logical relations between Aristotelian categorical propositions, Keynesian categorical propositions, and the interrelations between both? The main trouble is with contradictory relations, because the latter only holds with an arbitrary proposition A whose entire logical form is either affirmed, $+A$, or denied, $-A$. Thus affirming or denying each of the above literals is not enough to express contradictory relations between each of the available 128 categorical propositions. A way to revalidate the logical relations between Aristotelian categorical propositions has been proposed in [1], however, and we will extend the proposed rationale from affirmative to negative existential import. According to this rationale, there are 3 kinds of Aristotelian categorical propositions with respect to the issue of existential import about A:

(1) those where having existential import is made explicit in the proposition:
$\exists x Ax \wedge \mathscr{A}^4_C$

(2) those where lacking existential import is made explicit in the proposition:
$\neg\exists x Ax \wedge \mathscr{A}^4_C$

(3) those where having existential import is left implicit:
$\neg(\exists x Ax \wedge \neg\mathscr{A}^4_C)$

The latter case means the following: for any A^4_C, either existential import fails in it or nothing is said about it. This amounts to the disjunction $\neg\exists x Ax \vee \mathscr{A}^4_C$, which is equivalent to $\neg(\exists x Ax \wedge \neg\mathscr{A}^4_C)$. Given that (2) mostly leads to either inconsistencies or redundancies, the logical form of Aristotelian categorical propositions should be reduced to those relevant cases (1)-(3), thereby turning the first literal $\pm\exists x$ into a fixed component:

$$\mathscr{A}^4_{C_{\pm A \pm B}} = \exists x \pm Ax \wedge \pm\exists x \pm Bx \wedge \pm\exists x(\pm Ax \wedge \pm Bx)$$

There are only $2^6 = 64$ relevant categorical propositions with the remaining 6 literals, accordingly. In order to streamline the symbolization of categorical propositions for the rest of the paper, let us rewrite these as follows. For every predicate term $P = \{\pm A, \pm B\}$:

$\mathscr{A}^4_{C_{P!}}$ is the set of categorical propositions affirming the existence of $\pm P$s:
  $\mathscr{A}^4_{C_{P!}} = \exists x \pm Px \wedge A^4_C$
$\mathscr{A}^4_{C_{P?}}$ is the set of categorical propositions not affirming the existence of $\pm P$s:
  $\mathscr{A}^4_{C_{P?}} = \neg(\exists x \pm Px \wedge \neg A^4_C)$

The same process can be applied with respect to the issue of 'negative existential import' (whenever something is not-A), i.e. the set of categorical propositions $\mathscr{A}^4_{C_{\overline{A!}}}$. This yields a corresponding table of formulas for Keynesian categorical propositions, where the symbols from [1] and [12] are reworded correspondingly.

To summarize: formulas with explicit affirmative existential import are cases in which categorical propositions have import about A and are thereby limited to $6-1=5$ literals:

$$\mathscr{A}^4_{C_{A!}} = \exists xAx \wedge \pm\exists x \pm Bx \wedge \pm\exists x(\pm Ax \wedge \pm Bx)$$

By extension, propositions with explicit negative existential import about A are those which have import about not-A:

$$\mathscr{A}^4_{C_{\overline{A!}}} = \exists x \neg Ax \wedge \pm\exists x \pm Bx \wedge \pm\exists x(\pm Ax \wedge \pm Bx)$$

A generalized theory of categorical propositions consists in applying the same process to the 16 cases of ontological commitment about A and B, where the traditional issue of (affirmative) existential import includes 4 kinds of these logical forms.

TO is restored once a distinction is made between categorical propositions with or without explicit import, whereas the failure of *TO* arises where any of the four related propositions lacks explicit import. The result is that, instead of being invalidated, *TO* includes 3 valid logical squares with extended logical forms (with explicit or implicit import):

$S_1(A^4_C) = \{\mathbf{A}_{A!}, \mathbf{E}_{A!}, \mathbf{I}_{A?}, \mathbf{O}_{A?}\}$
$S_2(A^4_C) = \{\mathbf{A}_{A!}, \mathbf{E}_{A?}, \mathbf{I}_{A!}, \mathbf{O}_{A?}\}$
$S_3(A^4_C) = \{\mathbf{A}_{A?}, \mathbf{E}_{A!}, \mathbf{I}_{A?}, \mathbf{O}_{A!}\}$

3 complementary squares follow from such formulas and their characteristic bitstrings, isomorphically to the ones for Aristotelian categorical propositions:
$S_1(\mathscr{A}^4_C) = \{\mathbf{A}_{\overline{A!}}, \mathbf{E}_{\overline{A!}}, \mathbf{I}_{\overline{A?}}, \mathbf{O}_{\overline{A?}}\}$
$S_2(\mathscr{A}^4_C) = \{\mathbf{A}_{\overline{A!}}, \mathbf{E}_{\overline{A?}}, \mathbf{I}_{\overline{A!}}, \mathbf{O}_{\overline{A?}}\}$
$S_3(\mathscr{A}^4_C) = \{\mathbf{A}_{\overline{A?}}, \mathbf{E}_{\overline{A!}}, \mathbf{I}_{\overline{A?}}, \mathbf{O}_{\overline{A!}}\}$

The logical relations holding in these 3 squares may be checked by finding the characteristic bitstrings of their relata. An unrestricted model for any categorical proposition $\mathscr{A}^4_C$ is an ordered set of $2^4 = 16$ model sets relating $n = 2$ classes A and B, such that there can be a combination of 4 possible relations (i)–(iv) between elements of A and B: (i) some A is B, (ii) some A is not-B; (iii) some not-A is B; some not-A is not-B. The resulting 16 model sets are the following:

$w_1 = \{(i),(ii),(iii),(iv)\}$
$w_2 = \{(i),(ii),(iii)\}$
$w_3 = \{(i),(ii),(iv)\}$
$w_4 = \{(i),(iii),(iv)\}$
$w_5 = \{(ii),(iii),(iv)\}$
$w_6 = \{(i),(ii)\}$
$w_7 = \{(i),(iii)\}$
$w_8 = \{(ii),(iii)\}$
$w_9 = \{(i),(iv)\}$
$w_{10} = \{(ii)(,iv)\}$
$w_{11} = \{(iii),(iv)\}$
$w_{12} = \{(i)\}$
$w_{13} = \{(ii)\}$
$w_{14} = \{(iii)\}$
$w_{15} = \{(iv)\}$
$w_{16} = \{\}$

It follows from it that the 'internal' logical relations between Aristotelian categorical propositions hold after adding the clause of affirmative existential import about A in their logical form. The same does for the 'internal' relations between Keynesian categorical propositions, after assuming negative existential import about A. At the same time, the logical interrelations between Aristotelian and Keynesian categorical proposition still do not hold in some models. No wonder, since these interrelations hold only if existential assumption is made both about A and B. Whilst [1] and [12] dealt only with Aristotelian categorical propositions and existential import about A, a complete survey of the logical forms validating these interrelations requires a combination of 4 possible existential assumptions: affirmative and negative, on the one hand; about A or B, on the other hand.

For a comparative analysis of the categorical propositions that validate or invalidate *TO*, a comprehensive list of the 16 kinds of existential commitment can be made about their two predicate terms. The resulting 256 propositions, i.e. the $16 \times 8 = 128$ categorical propositions with explicit import $\mathscr{A}^4_{P!}$ together with their 128 contradictories $\mathscr{A}^4_{P?}$, may include from 0 to 4 kinds of existential assumption, whilst the aforementioned 'universal' assumptions (i.e. that either everything or nothing is P) merely amount to a lack of existential assumption. The resulting number of valid square is considerably extended, assuming that any proper square must include a set of four categorical propositions with two pairs of universals-existentials and affirmatives-negatives fulfilling logical requirements.

4 Conclusion and Prospects

Let us recapitulate the content of the present paper. Our aim was to extend the usual definition of categorical propositions in order to emphasize their logical form in terms of literals (both in TM and FOL). A generalized solution to the problem of existential import has been proposed beyond one former proposal by [1], including an introduction into a universal import for Keynesian propositions. The resulting semantics of bitstrings can be extended to any kinds of formulas, beyond the present of categorical propositions. An interesting case study is the set of knowledge statements considered in [2], whose general logical form

$$\pm @ \pm K \pm p$$

may be treated in the same pattern without introducing any intensional account of modal semantics. Another case is the set of dyadic relations, whose binary predicates of the form

$$\pm \exists x \pm \exists y \pm Rxy$$

can also be analyzed in a purely Boolean way.

Finally, the present paper needs some automatic process in order to determine the nature of logical relations between any pair of the available 256 categorical propositions. This requires the implementation of a programming machine in the style of Prolog, and this project will be pursued along with J.-Martín Castro-Manzano.

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