Alethe: Towards a Generic SMT Proof Format (extended abstract)

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The first iteration of the proof format used by the SMT solver veriT was presented ten years ago at the first PxTP workshop. Since then the format has matured. veriT proofs are used within multiple applications, and other solvers generate proofs in the same format. We would now like to gather feedback from the community to guide future developments. Towards this, we review the history of the format, present our pragmatic approach to develop the format, and also discuss problems that might arise when other solvers use the format.

Over the years the production of machine-consumable formal proofs of unsatisfiability from SMT solvers [6] has attracted significant attention [4]. Such proofs enable users to certify unsatisfiability results similarly to how satisfiable results may be certified via models. However, a major difficulty that SMT proof formats must address is the complex and heterogeneous nature of SMT solvers: a SAT solver drives multiple, often very different, theory solvers; instantiation procedures generate ground instances; and heavily theory-dependent simplification techniques ensure that the solvers are fast in practice. Moreover, how each of these components works internally can also differ from solver to solver. As a testament to these challenges proof formats for SMT solvers have been mostly restricted to individual solvers and no standard has emerged. To be adopted by several solvers, an SMT proof format must be carefully designed to accommodate needs of specific solvers. This will require repeated refinement and generalization.

The basis for our efforts in this field is the proof format implemented by the SMT solver veriT [8] that is now mature and used by multiple systems. To further improve the format, as well as to accommodate not only the reasoning of the SMT solver veriT but also of other solvers, we are currently extending the format and developing better tooling, such as an independent proof checker. To facilitate this effort and overall usage, we are also writing a full specification. To emphasize the independence of the format we are baptizing it *Alethe*.[1] We do not presume to propose a standard format a priori. Instead we believe that Alethe, together with its tooling, can provide a basis for further discussions on how to achieve a format to be used by multiple solvers.

1 The State of Alethe

Alethe combines two major ideas whose roots reach back ten years to the first PxTP workshop in 2011 [7, 9]. It was proposed as an easy-to-produce format with a term language very close to SMT-LIB [5], the standard input language of SMT solvers, and rules with a varying level of granularity, allowing implicit

[1] Alethe is a genus of small birds and the name resembles *aletheia*, the Greek word for truth.

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proof steps in the proof and thus relying on powerful proof checkers capable of filling the gaps. Since then the format has been refined and extended [2]. It is now mature, supports coarse- and fine-grained proof steps capturing SMT solving for the SMT-LIB logic UFLIRA[2] and can be reconstructed by the proof assistants Coq [1, 11] and Isabelle [12, 14]. In particular, the integration with Coq was also used as a bridge for the reconstruction of proofs from the SMT solver CVC4 [3] in Coq, where its proofs in the LFSC format [15] were first translated into the veriT format before reconstruction. Finally, the format will also be natively supported in the upcoming cvc5. solver[3]

On the one hand, Alethe uses a natural-deduction style calculus driven mostly by resolution [7]. To handle first-order reasoning, dedicated quantifier instantiation rules are used [9]. On the other hand, it implements novel ideas to express reasoning typically used for processing, such as Skolemization, renaming of variables, and other manipulations of bound variables [2]. While the format was always inspired by the SMT-LIB language, we recently [12] changed the syntax of Alethe to closely resemble the command structure used within SMT-LIB. When possible Alethe uses existing SMT-LIB features, such as the `define-fun` command to define constants and the `:named` annotation to implement term sharing.

The following proof fragment gives a taste of the format. The fragment first renames the bound variable in the term $\exists x. f(x)$ from x to vr and then skolemizes the quantifier. A proof is a list of commands. The *assume* command introduces an assumption, *anchor* starts a subproof, and *step* denotes an ordinary proof step. Steps are annotated with an identifier, a rule, and premises. The SMT-LIB command *define-fun* defines a function. The rule *bind* used by step `t1` performs the renaming of the bound variable. It uses a subproof (Steps `t1.t1` and `t1.t2`). The subproof uses a context to denote that `x` is equal to `vr` within the subproof. The `anchor` command starts the subproof and introduces the context. The `bind` rule does not only make it possible to rename bound variables, but within the subproof it is possible to simplify the formula as done during preprocessing. The steps `t2` and `t3` use resolution to finish the renaming. In step `t4` the bound variable is skolemized. Skolemization uses the choice binder ε and derives $f(\varepsilon vr. f(vr))$ from $\exists vr. f(vr)$. To simplify the reconstruction the choice term is introduced as a defined constant (by `define-fun`). Finally, resolution is used again to finish the proof.

```
(assume a0 (exists ((x A)) (f x)))
(anchor :step t1 :args (:= x vr))
(step t1.t1 (cl (= x vr)) :rule cong)
(step t1.t2 (cl (= (f x) (f vr))) :rule cong)
(step t1 (cl (= (exists ((x A)) (f x))
                (exists ((vr A)) (f vr)))) :rule bind)
(step t2 (cl (not (= (exists ((vr A)) (f x))
                     (exists ((vr A)) (f vr))))
             (not (exists ((vr A)) (f x)))
             (exists ((vr A)) (f vr))) :rule equiv_pos1)
(step t3 (cl (exists ((vr A)) (f vr))) :premises (a0 t1 t2) :rule resolution)
(define-fun X () A (choice ((vr A)) (f vr)))
(step t4 (cl (= (exists ((vr A)) (f vr)) (f X))) :rule sko_ex)
(step t5 (cl (not (= (exists ((vr A)) (f vr)) (f X)))
             (not (exists ((vr A)) (f vr)))
             (f X)) :rule equiv_pos1)
(step t6 (cl (f X)) :premises (t3 t4 t5) :rule resolution)
```

[2]That is the logic for problems containing a mix of any of quantifiers, uninterpreted functions, and linear arithmetic.
[3]`https://cvc4.github.io/2021/04/02/cvc5-announcement.html`

The output of Alethe proofs from veriT has now reached a certain level of maturity. The 2021 version of the Isabelle theorem prover was released earlier this year and supports the reconstruction of Alethe proofs generated by veriT. Users of Isabelle/HOL can invoke the `smt` tactic. This tactic encodes the current proof goal as an SMT-LIB problem and calls an SMT solver. Previously only the SMT solver Z3 was supported. Now veriT is supported too. If the solver produces a proof, the proof is reconstructed within the Isabelle kernel. In practice, users will seldom choose the `smt` tactic themselves. Instead, they call the Sledgehammer tool that calls external tools to find relevant facts. Sometimes, the external tool finds a proof, but the proof cannot be imported into Isabelle, requiring the user to write a proof manually. The addition of the veriT-powered `smt` tactic halves [14] the rate of this kind of failures. The improvement is especially pronounced for proofs found by CVC4. A key reason for this improvement is the support for the conflicting-instance instantiation technique within veriT. Z3, the singular SMT solver supported previously, does not implement this technique. Nevertheless, it is Alethe that allowed us to connect veriT to Isabelle, and we hope that the support for Alethe in other solvers will ease this connection between powerful SMT solvers and other tools in the future.

The process of implementing proof reconstruction in Isabelle also helped us to improve the proof format. We found both, possible improvements in the format (like providing the Farkas' coefficient for lemmas of linear arithmetic) and in the implementation (by identifying concrete errors). One major shortcoming of the proofs were rules that combined several simplification steps into one. We replaced these steps by multiple simple and well-defined rules. In particular every simplification rule addresses a specific theory instead of combining them. An interesting observation of the reconstruction in Isabelle is that some steps can be skipped to improve performance. For example, the proofs for the renaming of variables are irrelevant for Isabelle since this uses De Bruijn indices. This shows that reconstruction specific optimizations can counterbalance the proof length which is increased by fine-grained rules. We will take this prospect into account as we further refine the format.

2 A Glance Into the Future

The development of the Alethe proof format so far was not a monolithic process. Both practical considerations and research progress — such as supporting fine-grained preprocessing rules — influenced the development process. Due to this, the format is not fully homogeneous, but this approach allowed us to quickly adapt the format when necessary. We will continue this pragmatic approach.

Speculative Specification. We are writing a speculative specification.[4] During the development of the Isabelle reconstruction it became necessary to document the proof rules in a coherent and complete manner. When we started to develop the reconstruction there was only an automatically generated list of rules with a short comment for each rule. While this is enough for simple tautological rules, it does not provide a clear definition of the more complex rules such as the linear arithmetic rules. To rectify this, we studied veriT's source code and wrote an independent document with a list of all rules and a clear mathematical definition of each rule. We chose a level of precision for these descriptions that serves the implementer: precise enough to clarify the edge case, but without the details that would make it a fully formal specification. We are now extending this document to a full specification of the format. This specification is speculative in the sense that it will not be cast in stone. It will describe the format as it is in use at any point in time and will develop in parallel with practical support for the format within SMT solvers, proof checkers, and other tools.

[4]The current version is available at `http://www.verit-solver.org/documentation/alethe-spec.pdf`.

Flexible Rules. The next solver that will gain support for the Alethe format is the upcoming cvc5 solver. Implementing a proof format into another solver reveals where the proof format is too tied to the current implementation of veriT. On the one hand, new proof rules must be added to the format — e.g., veriT does not support the theory of bitvectors, while cvc5 does. When CVC4 was integrated into Coq via a translation of its LFSC proofs into Alethe proofs [11], an ad-hoc extension with bitvector rules was made. A revised version of this extension will now be incorporated into the upcoming specification of the format so that cvc5 bitvector proofs can be represented in Alethe. Further extensions to other theories supported by cvc5, like the theory of strings, will eventually be made as well.

Besides new theories, cvc5 can also be stricter than veriT in the usage of some rules. This strictness can simplify the reconstruction, since less search is required. A good example of this is the `trans` rule that expresses transitivity. This rule has a list of equalities as premises and the conclusion is an equality derived by transitivity. In principle, this rule can have three levels of "strictness":

1. The premises are ordered and the equalities are correctly oriented (like in cvc5), e.g., $a = b$, $b = c$, and $c = d$ implies $a = d$.
2. The premises are ordered but the equalities might not be correctly oriented (like in veriT), e.g., $b = a$, $c = b$, and $d = c$ implies $d = a$.
3. Neither are the assumptions ordered, nor are the equalities oriented, e.g., $c = b$, $b = a$, and $d = c$ implies $d = a$.

The most strict variant is the easiest to reconstruct: a straightforward linear traversal of the premises suffices for checking. From the point of view of producing it from the solver, however, this version is the hardest to implement. This is due to implementations of the congruence closure decision procedure [13, 10] in SMT solvers being generally agnostic to the order of equalities, which can lead to implicit reorientations that can be difficult to track. Anecdotally, for cvc5 to achieve this level of detail several months of work were necessary, within the overall effort of redesigning from scratch CVC4's proof infrastructure. Since we cannot assume every solver developer will, or even should, undertake such an effort, all the different levels of granularity must be allowed by the format, each requiring different complexity levels of checking.

To keep the proof format flexible and proofs easy to produce, we will provide different versions of proof rules, with varying levels of granularity as in the transitivity example case above, by *annotating* them. This leverages the rule *arguments*, which are already used by some rules. For example, the Farkas' coefficient of the linear arithmetic rule are provided as arguments. This puts pressure on proof checkers and reconstruction in proof assistants to support all the variants or at least the most general one (at the cost of efficiency). Hence, our design principle here is that the annotation is optional: the absence of an annotation denotes the least strict version of the rule.

Powerful Tooling. We believe that powerful software tools may greatly increase the utility of a proof format. Towards this end we have started implementing an independent proof checker for Alethe. In contrast to a proof-assistant-based reconstruction, this checker will not be structured around a small, trusted kernel, and correct-by-construction extensions. Instead, the user would need to trust the implementation does not lead to wrong checking results. Instead, its focus is on performance, support for multiple features and greater flexibility for integrating extensions and refinements to the format. The Isabelle checker is currently not suited to this task — one major issue is that it does not support SMT-LIB input files.[5]

[5] A version capable of doing so was developed for Z3 but it was unfortunately lost.

This independent checker will also serve as a proof "elaborator". Rather than checking, it will also allow converting a coarse-grained proof, containing implicit steps, to a fine-grained one, with more detailed steps. The resulting proof can then be more efficiently checked by the tool itself or via proof-assistant reconstructions. An example of such elaboration is the transitivity rule. If the rule is not in its most detailed version, with premises in the correct order and none implicitly reordered, it can be elaborated by greedily reordering the premises and adding proof steps using the symmetry of equality. Note however that in the limit detailing coarse-grained steps can be as hard as solving an SMT problem. Should such cases arise, the checker will rely on internal proof-producing procedures capable of producing a detailed proof for the given step. At first the veriT and cvc5 solvers, which can produce fine-grained proofs for several aspects of SMT solving, could be used in such cases.

A nice side effect of the use of an external checker is that it could prune useless steps. Currently SMT solvers keep a full proof trace in memory and print a pruned proof after solving finishes. This is in contrast to SAT solvers that dump proofs on-the-fly. For SAT proofs, the pruned proof can be obtained from a full trace by using a tool like DRAT-TRIM. There is some ongoing work by Nikolaj Bjørner on Z3 to also generate proofs on-the-fly, but it is not clear how to support preprocessing and quantifiers.[6]

3 Conclusion

We have presented on overview of the current state of the Alethe proof format and some ideas on how we intend to improve and extend the format, as well as supporting tools. In designing a new proof format supported across two solvers we hope to provide a first step towards a format adopted by more solvers. This format allows several levels of detail, and is thus flexible enough to reasonably easily produce proofs in various contexts. We intend to define a precise semantics at each level though. This distinguishes our format from other approaches, such as the TSTP format [16], that are probably easier to adopt but only specify the syntax, leading to very different proofs generated by the various provers supporting it.

One limit of our approach for proofs is that we cannot express global transformations like symmetry breaking. SAT solvers are able to add clauses (DRAT clauses) such that the overall problems is equisatisfiable. It is unclear however how to add such clauses in the SMT context.

Overall, we hope to get feedback from users and developers to see what special needs they have and exchange ideas on the proof format.

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