# **ArGot:** A Glossary of Terms extracted from the arXiv

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We introduce ArGoT, a data set of mathematical terms extracted from the articles hosted on the arXiv website. A term is any mathematical concept defined in an article. Using labels in the article's source code and examples from other popular math websites, we mine all the terms in the arXiv data and compile a comprehensive vocabulary of mathematical terms. Each term can be then organized in a dependency graph by using the term's definitions and the arXiv's metadata. Using both hyperbolic and standard word embeddings, we demonstrate how this structure is reflected in the text's vector representation and how they capture relations of entailment in mathematical concepts. This data set is part of an ongoing effort to align natural mathematical text with existing Interactive Theorem Prover Libraries (ITPs) of formally verified statements.

#### **1** Introduction and Motivation

Mathematical writing usually adheres to strict conventions of rigor and consistent usage of terminology. New concepts are usually introduced in characteristically worded definitions (with patterns like *if and only if* or *we say a group is abelian...*). This feature can be used to train language models to detect if a term is defined in a text. Using this, we have created ArGoT (**arXiv Glossary of Terms**), a silver standard data set of terms defined in the Mathematical articles of the arXiv website. We showcase several interesting applications of this data. The data set includes the articles and paragraph number in which each term appears. By using article metadata, we show that this can be an effective way of assigning an arXiv mathematical category<sup>1</sup> to each term. Another application is to join the terms with more than one word into a single token. These phrases usually represent important mathematical concepts with a specific meaning. We show how standard word embedding models like word2vec [13] and GloVe [16] capture this by embedding phrases instead of individual words. Even more, the word-vector can be used to predict which mathematical field the term belongs to, and hypernimity relations.

All these properties makes ArGoT a data set that will be of interest to the broader NLP research community by providing abundant examples for automated reasoning and NLU systems. Our main objective is to organize a comprehensive dependency graph of mathematical concepts that can be aligned with existing libraries of formalized mathematics like mathlib.<sup>2</sup> The data is downloadable from https:// sigmathling.kwarc.info/resources/argot-dataset-2021/ and the all the code that went into producing it is in: https://github.com/lab156/arxivDownload

This data set was created as part of the Formal Abstracts project. Our group has benefited from a grant from the Sloan Foundation (G-2018-10067) and from the computing resources startup allocation #TG-DMS190028 and #TG-DMS200030 on the Bridges-2 supercomputer at the Pittsburgh Supercomputing Center (PSC).

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arXiv's categories within mathematics: https://arxiv.org/archive/math

<sup>&</sup>lt;sup>2</sup>https://github.com/leanprover-community/mathlib

Term	Count
lie algebra	20524
hilbert space	16881
function	14920
banach space	14461
metric space	12882
_inline_mathmodule	12731
topological space	12518
disjoint union	11436
vector space	11337
simplicial complex	10943

Table 1:Most common multiword entries inthe data base.

Classification Task							
Method	Precision	Recall	F1				
SGD-SVM	0.88	0.87	0.87				
Conv1D	0.92	0.92	0.92				
BiLSTM	0.93	0.93	0.93				
NER Task							
ChunkParse	0.32	0.68	0.43				
LSTM-CRF	0.69	0.65	0.67				

Table 2: Training metrics on the classificationand NER tasks.

### 2 Description of the Term-Definition Extraction Method

In [2, 9], the authors describe the method used to obtain the training data for a text classification model that identifies definitions and the Named Entity Recognition (NER) model that identifies the term being defined.

The classification task consists of training a binary classifier to determine whether a paragraph is a definition or not. We use the  $\begin{definition}...\end{definition}$  in the article's IATEX source to identify true examples. To gather non-definitions, we randomly sample paragraphs out of the same articles. The source of the training data is the IATEX source code of the articles available from the arXiv website. A total of 1,552,268 paragraphs labeled as definitions or non-definitions were produced for training. It was split as follows: 80% training 10% testing and 10% validation. This data was used to train three different and common classification models:

- The Stochastic Gradient Descent with Support Vector Machines (SGD-SVM).
- The one-dimensional convolutions (Conv1D) neural network.
- And Bidirectional LSTM (BiLSTM).

For the first method, we used the implementation distributed with scikit-learn library [15]. The last two were implemented in Tensorflow. Table 2 shows the most common metrics of performance for each method.

The definitions are then fed into a NER model to identify the term being defined in them. The data used to train the NER model comes from the Wikipedia English dump<sup>3</sup> and several mathematical websites like PlanetMath<sup>4</sup> and The Stacks Project.<sup>5</sup>

We tested two different implementations of the NER system, the first is the *ChunkParse* algorithm available from the NLTK library [3]. The second is a time-distributed LSTM (LSTM-CRF) [11]. Both architectures use a similar set of features that in addition to the words that form the text, detect if the word is capitalized, its part-of-speech (POS) and parses punctuation e.g. to tell if a period is part of an abbreviation or an end of line. To compare the two implementations, we used the ChunkScore method in the NLTK library [3]. The results appear in Table 2.

<sup>&</sup>lt;sup>3</sup>https://dumps.wikimedia.org/

<sup>&</sup>lt;sup>4</sup>https://planetmath.org/

<sup>&</sup>lt;sup>5</sup>https://stacks.math.columbia.edu/

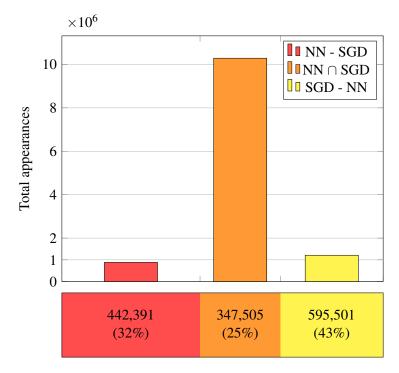


Figure 1: Comparison of the two glossaries. The bar graph on top counts the total appearances of a term in both the NN and SGD glossaries. The bottom compares the relative sizes of the NN-only, intersection, and SGD-only distinct terms.

We have compiled two different and independent glossaries by running the algorithm through all of the arXiv's mathematical content. The first one is based on neural networks (NN), it uses LSTM for both the classification and NER tasks. In contrast, the second one combines the SGD and ChunkParser method to provide a completely independent approach to the previous model.

It is interesting to compare the results obtained using the two models. For the classification task, we have observed Cohen's kappa ( $\kappa$ ) inter-rater agreement of 93% between the results produced by the two methods. This corresponds to a high degree of agreement between the two classifiers [4].

As for the final results, Figure 1 compares the two glossaries by counting the number of times a term appears in either glossary, and the number of distinct terms. The results point to a high consistency of the two systems on a relatively small set of 350,000 terms.

Table 1 lists some of the most frequently found terms in the data set.

#### 2.1 Format and Design of the Data Set

The ArGoT data set is distributed in the form of compressed XML files that follow the same naming convention the arXiv's bulk download distribution.<sup>6</sup> For instance, Table 3 shows a sample entry in the fifth file corresponding to July, 2014. The definition's statement and terms (definiendum) are specified in the stmnt and dfndum tags respectively and the paragraph index is specified as an attribute of the definition tag.

<sup>&</sup>lt;sup>6</sup>arXiv Bulk Data Access: https://arxiv.org/help/bulk\_data

```
<article name="1407_005/1407.2218/1407.2218.xml" num="89">
<definition index="51">
        <stmnt> Assume _inline_math_. We define the following space-time
        norm if _inline_math_ is a time interval _display_math_ </stmnt>
        <dfndum>space-time norm</dfndum>
</definition>
<//article>
```

Table 3: Example of an entry in the term's data set. The statement of the definition is contained in the <stmnt> tag. The terms (definiendum) are listed as <dfndum> tags. Each entry contains all the information to recover, article's name and paragraph's position.

Category:	Count	Category:	Count
math.FA	5922	math.GN	108
math.AP	2045	math.RT	85
math.PR	1022	math.SG	77
math.DS	833	math.GT	76
math.OA	595	math.CO	61
math.CA	535	math.ST	61
math.DG	483	math.KT	50
math-ph	466	math.GM	48
math.OC	398	math.AG	35
math.CV	304	math.RA	33
math.NA	275	math.HO	32
math.GR	226	math.CT	23
math.MG	173	math.AT	15
math.LO	168	math.QA	10
math.SP	163	math.AC	8
math.NT	131		

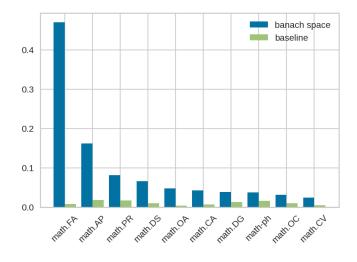
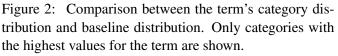


Table 4: Category profile for the term: *Banach Space*. The codes are part of the metadata for each arXiv article.



### 3 Augmenting Terms with arXiv's Metadata

Each mathematical article in the arXiv is classified in one or more *categories* by the author at the time of submission. Categories include math.FA and math.PR which stand for Functional Analysis and Probability respectively. The full list is available at https://arxiv.org/archive/math. This is part of the arXiv's metadata and also records information like the list of authors, math subject classification (MSC) codes, date of submission, etc.

By counting the categories in which a certain term is used, we get an idea of the subjects that it belongs to. In Table 4, we see the category profile of a very common term. Since the number of articles in each category varies significantly, we also take into account the baseline distribution, that is, the ratio of articles in each category to the total number of articles. Hence, it is possible to give an empirical score of a term's pertinence to a certain category by comparing its category profile with the baseline distribution. In order to measure how much of an outlier a term is to the baseline distribution, we use the KL-divergence:

$$D_{\mathrm{KL}}(P||Q) = \sum_{x \in X} P(x) \log(P(x)/Q(x)),$$

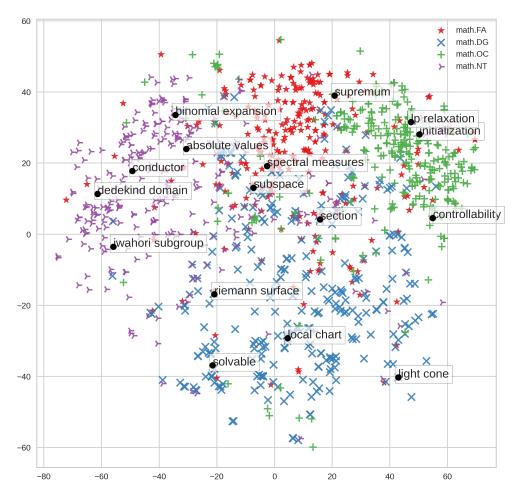


Figure 3: t-SNE visualization of the word vectors of selected terms in the data set. The terms are selected to be specific to the four categories in the picture. Points with a label are selected at random.

where P and Q are the probability distributions of the term and the baseline respectively. And, X is the set of all the categories.

The next step is to generate word embeddings. To prepare for this, we modify the text by joining multiword terms in ArGoT to produce individuals tokens. After normalizing the text, i.e. converting to lowercase and removing punctuation and special characters; the result is a large amount of text that is ready to be consumed by either the word2vec or GloVe algorithms. In Figure 3, we observe a t-SNE (t-distributed stochastic neighbor embedding) visualization of a word2vec model produced this way. In this image, each term is assigned its most frequent category. Notice that even though the ArGoT data set has no access to the arXiv categories, the vectors in the same category cluster together. We consider this as a strong indication of alignment between clusters and categories.

## 4 Using Hyperbolic Word Embeddings to Extract Hypernymy Relations

It is natural to want to organize mathematical concepts into taxonomies of various sorts. For instance, the SMGloM project [8] introduced a rich standard for mathematical ontologies. Another approach aims

to create a semantic hierarchy of concepts such that for a given term we can enumerate all its hypernyms [18].

This can be achieved by counting the co-occurrence [10] of terms in definitions. This approach has certain drawbacks, for instance, it relies on co-occurrence examples for each pair of terms, this ends up producing an abundance of disconnected (i.e. not co-occurring) terms [1].

Another possibility, involves the use of *hyperbolic word embeddings*, in this setting the hypernimity relation becomes a geometric vector in hyperbolic space. This implies that every two terms in the embedding can be compared by using the hyperbolic metric. This type of word embeddings is known to outperform euclidean models in the representation of hierarchical structures [14].

We used PoincareGlove [17] to create hyperbolic word embeddings. This algorithm modifies the GloVe *euclidean* objective function to use a hyperbolic metric instead. In addition to the same text input as word2vec and GloVe, this model requires a small set of examples in order to interpret the embedding. For general purpose English text, WordNet [6] is the standard choice. In WordNet, every entry is assigned an integer level in a hypernymy hierarchy (this is the max\_depth attribute of the NLTK's WordNet API).<sup>7</sup>

To generate something analogous to WordNet levels for mathematical content, we opted for the PlanetMath data set. This is due to its relatively small size, broad coverage of mathematical knowledge and independence of the arXiv data. Given two term-definition pairs  $(t_1, D_1)$  and  $(t_2, D_2)$ , we say that term  $t_2$ *depends* on the term  $t_1$  if  $D_2$  contains  $t_1$ . For small sets of term-definition pairs with no interdependence, this simple criterion is enough to create a directed graph (V, E) where V is the set of all the terms and E is the set of all the dependency relations. To assign a level  $\lambda(v)$  to every vertex  $v \in V$ , solve the following integer linear program:

$$\min \sum_{(v,w)\in E} \lambda(w) - \lambda(v), \quad \text{such that} \quad \lambda(w) - \lambda(v) \ge 1 \quad \forall (v,w) \in E.$$

This linear model appears in [7] as a subtask of a directed graph drawing algorithm. There, it is used to estimate the ideal number of levels to draw a directed graph.

Table 5 shows the nearest neighbors of four different terms. The neighbors are found using the Euclidean distance. The terms are sorted in order of the average value of their *y*-coordinates (which in the upper-half plane model represents the variance of the underlying Gaussian distribution). This is referred to as the IS-A rating.

#### **5** Conclusions and Further Work

We introduced ArGoT, an comprehensive glossary of mathematics automatically collected from the mathematical content on the arXiv website. Essentially, it is set of term-definition pairs, where each pair can be contextualized in a large semantic network of mathematical knowledge, i.e., dependency graph. We also showed how this network is reflected in the latent space of its vector embeddings. This has great potential for use in experimentation of natural language algorithms, by providing a source of logically consistent data.

This project is an ongoing effort to align mathematical concepts in natural language with online repositories of formalized mathematics like mathlib.<sup>8</sup> As described in [12], this type of alignment is called *automatically found alignent*.

<sup>&</sup>lt;sup>7</sup>https://www.nltk.org/howto/wordnet.html

<sup>&</sup>lt;sup>8</sup>https://github.com/leanprover-community/mathlib

Term	IS-A	Term	IS-A
hyperbolic_metric	-1.11		
euclidean_metric	-0.59	digraph	-0.51
metrics	-0.58	undirected_graph	-0.35
riemannian_metric	-0.46	undirected	-0.20
riemannian	-0.42	directed_graph	0.0
riemannian_manif	-0.40	graph	1.24
curvature	-0.27		
metric	0.0		
banach_algebra	-1.11	probability_distr	-0.24
normed_space	-0.98	random_variable	0.0
banach_spaces	-0.38	expectation	0.23
banach	-0.29	distribution	0.46
closed_subspace	-0.25	probability	0.67
banach_space	0.0		
norm	0.79		

Table 5: Query results sorted by IS-A score (terms in upper lines tend to depend semantically on lower lines). Cosine similar words were sorted by the IS-A rating of the term in bold font.

In the near future we plan to further improve on the classification and NER tasks by creating a data set using solely the neural version of the classifier and NER model. Also, by using state-of-the-art methods like the masked transformer language model [5] to further improve the results. We also plan to compile the complete dependency graph in one large graph database.

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