# Solving Some Geometry Problems of the Náboj 2023 Contest with Automated Deduction in GeoGebra Discovery 

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#### Abstract

In this article, we solve some of the geometry problems of the Náboj 2023 competition with the help of a computer, using examples that the software tool GeoGebra Discovery can calculate. In each case, the calculation requires symbolic computations. We analyze the difficulty of feeding the problem into the machine and set further goals to make the problems of this type of contests even more tractable in the future.


## 1 Introduction

With the everyday rise of Artificial Intelligence (AI), the power of computers has become tangible for the masses. Yes, it can do your homework (not just in maths), but it can also pass your A-level exams ${ }^{1}$ A long series of ad-hoc studies have shed light on what the present can offer: often instant and perfect answers to questions that take years of learning to solve by human means. This raises a number of research questions, such as whether the current school system is still needed, whether teachers are still needed, or whether it is enough to have $\left.\mathrm{AI}\right|^{2}$ Of course, alongside the praise, there are also many criticisms: AI sometimes makes mistakes, especially in textual problem settings where the question is formulated in a challenging way.

Automatic geometrical derivations, on the other hand, are perfect and, as such, there is no such a major possibility of error. The answer is not derived from a statistically computed result (as is so often the case with AI-based algorithms), but a verifiable derivation is given in each case. We do not claim that the two directions cannot meet once, and indeed, ultimately, AI should refer to, i.e. use, the ADG algorithm as a subroutine. There are already prototypes working in this direction, e.g. the WolframAlpha system has been successfully coupled with an AI frontend 3

[^0]P. Quaresma and Z. Kovács (Ed.): Automated Deduction in Geometry 2023 (ADG 2023).
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In this contribution, we aim for less. We are just trying to solve competitive problems with an ADG algorithm in the background. However, we leave the exact task setting to the user. This means that it is up to the user to provide the exact flow of the editing task with concrete steps. This must be done in GeoGebra Discovery ${ }^{4}$. However, for the inference, which requires a symbolic calculation in the tasks, the ADG algorithm steps in and, as we will see, gives the correct result in all cases. In the second half of the paper, we propose how the range of problems that can be solved in this way can be further extended.

## 2 The Náboj Contest

According to naboj.org Náboj is an international mathematical competition designed for teams of five high-school students that represent their schools, which lasts 120 minutes and where they are trying to solve as many given problems as possible. As soon as the team correctly solves any of the problems, they receive new ones. The solutions of the problems are usually numerical. The team that solves most problems correctly in the given time limit wins. The Náboj problems in contrast to the most school exercises require a certain amount of inventiveness and ingenuity.

Traditionally, many geometric tasks require proving. Checking if a proof is correct may be a difficult process for the organizers, so it is usually avoided to set proof related problems during contests like Náboj. Instead, problem settings require computing fractions, or better, providing a non-trivial algebraic number. As a consequence, geometric problems in Náboj are mostly non-geometric, or if still so, they are set in a way to require a numerical result.

All the problems we discuss in this paper will have exact answers, non-trivial fractions or some root expressions. It is clear that the exact definition of the latter requires symbolic computation. By default, software that allows a geometric problem to be well visualized (GeoGebra in particular) provides only numerical support for measuring the quantity in question. However, the software presented here, the GeoGebra fork GeoGebra Discovery, is capable of making measurements symbolically. This also means that a full proof has already been created in the background, but the user is not informed about this.

The use of electronic aids in the Náboj competition has recently been restricted since the competition is on-site again. In the long term, electronic assistance will certainly not be eliminated. It is a fact that students are turning to AI for quick help, and the tasks set must take this into account. It may seem like fun, but the rapid pace of the modern age also poses a huge challenge for assignment writers: is the task set difficult enough to prevent the AI from giving a quick, accurate, complete answer? But the task setter is not only fighting the AI, but also the ADG algorithm: cannot the task be solved in a flash if the right data is entered into the right software in the right way and the right button is pressed?

Overall, we conclude that the tasks set should be tested with different software before they are announced, in order to avoid embarrassing surprises. Even if we manage to keep the students working with paper and pencil only for the 2 hours of the competition, i.e. to exclude electronic assistance, it raises serious questions about what the AI and ADG algorithms can achieve with the tasks set. In the preliminary analysis of problems, but also in retrospect, when we return to the correct solution of problems in mathematics class or in a specialised course, it may be useful to use the electronic method.

[^1]Problem 6. The rhombus flower grows according to the following pattern: In the middle there is a square blossom with two diagonals of length 1 . In the first step the horizontal diagonal is doubled creating a new quadrilateral. In the next step the vertical diagonal is doubled and again a new quadrilateral blossom is generated. This procedure is continued until there is a flower with five quadrilateral blossoms. Find the perimeter of the outer (i.e. the fifth) blossom.


Result. $\quad 8 \sqrt{2}$
Solution. The fifth blossom is a square with diagonals of length 4 , hence the length of its side is $2 \sqrt{2}$ and the perimeter equals $8 \sqrt{2}$.

Figure 1: Problem setting 6 and the official solution of Náboj 2023

## 3 Mathematical Background

The method used by GeoGebra Discovery is essentially the Recio-Vélez method [1], complemented by the algorithm given in [2]. For the problems of the Náboj competition we are discovering a ratio of two lengths, it is therefore worth using an elimination method.

As an illustration, we show how the program solves Problem 6 of Náboj 2023 (Fig. (1).
When drawing the figure in GeoGebra Discovery (Fig. 2), we learn that the problem can be simplified to three squares. First, an arbitrary square $A B C D$ is drawn. Then midpoint $E$ of $A C$ is defined. This point will be then reflected about $A$ to get $E^{\prime}$ and about $B$ to get $E_{1}^{\prime}$. Now a second square $E^{\prime} E_{1}^{\prime} F G$ is drawn. Finally, by reflecting $E$ about $E^{\prime}$ and $E_{1}^{\prime}$ we get points $E_{2}^{\prime}$ and $E_{3}^{\prime}$, respectively, and create the square $E_{2}^{\prime} E_{3}^{\prime} H I$ as well. By defining $s=A C, t=E_{2}^{\prime} E_{3}^{\prime}$ and $P=4 t$, we can use GeoGebra Discovery's Relation tool to compare $s$ and $P$ and we learn (after pressing the button "More..." to obtain a symbolic analysis) that $P=8 \sqrt{2} \cdot s$. The report of the symbolic analysis of Problem 6 in GeoGebra Discovery shows that "It is generally true that: $P=(8 \sqrt{2}) \cdot s$ under the condition: the construction is not degenerate" (see Fig. 3). The construction steps in GeoGebra Discovery for Problem 6 can be taken from Table 1. (These extra explanations are listed in the Appendix.)

Here we highlight that the problem setting could be further simplified by skipping the construction of the two latter squares. In fact, only the reflection points matter. Also, we used the fact that the perimeter of a square equals to four times the length of a side, but this piece of information could have been ignored and asked the program to learn this on its own.

At this point, we will jump to the last steps and assuming that it is possible to do that without loss of generality, GeoGebra Discovery, in the background (in an invisible way for the normal user, but in a verifiable way via its debug messages), decides to substitute $A=(0,0)$. This will simplify the computations and the final question is if $e_{21}: m \cdot v_{24}=4 v_{23}$ holds for a given value of $m$. This can be answered by eliminating all variables but $m$ from all of the equations $e_{1}, e_{2}, \ldots, e_{14}, e_{18}, \ldots, e_{21}$, and we learn that $m^{2}=128$. That is, by assuming that $m>0$, we obtain that $m=8 \sqrt{2}$.

The algebraic solution is, of course, quite complicated. Also, the way used in GeoGebra Discovery by constructing all the required objects, may still be very complicated when compared to the quick official solution.


Figure 2: Sketching Problem 6 in GeoGebra Discovery

## 4 Problems that can be Solved with GeoGebra Discovery

In this section we list four additional problems that can be solved with GeoGebra Discovery, assuming some effort. In fact, some other Náboj 2023 problems can be supported as well, but they may require some additional steps. See the next section for more details.

### 4.1 Pentominos (Problem 15)

The problem setting can be seen in Fig. 4
At a first look, it seems complicated to draw a figure that describes the problem setting adequately. Some attempts may lead to Fig. 5. lines $g_{1}=E K$ and (after extending the large pentomine with square $D I N F) l_{1}=H N$ help finding point $O$. Then, segment $m_{1}=E O$ will be one side of the small $X$-pentomino, and it will be possible to compare it to one of the sides of the large pentomino. For more details about the construction steps have a look at Table 2 .

Now, asking the relation between $m_{1}$ and $f$ we obtain that $f=\frac{1}{2} \cdot \sqrt{10} \cdot m_{1}$. Even if GeoGebra Discovery cannot compare areas symbolically in a direct way, we can still conclude that the ratio of the


Figure 3: Report of a symbolic analysis of Problem 6 in GeoGebra Discovery

Problem 15. Antonia drew a small $X$-pentomino made of 5 congruent squares. Then she drew two perpendicular diagonals of this pentomino with dotted lines. Finally she constructed a bigger $X$-pentomino with some of the sides lying on the diagonals of the small pentomino as in the figure. Find the ratio of the area of Antonia's big pentomino to the area of the small one.


Result. 5: 2.
Figure 4: Problem setting 15


Figure 5: Problem setting 15 in GeoGebra Discovery

Problem 23. The legs of a right-angled triangle have lengths 11 and 23. A square of side length $t$ has two of its sides lying on the legs of the triangle and one vertex on its hypotenuse as in the picture. Find $t$.


Result. $\frac{253}{34}$
Figure 6: Problem setting 23
areas must be

$$
f^{2}: m_{1}^{2}=10: 4=5: 2
$$

We remark here that a sophisticated way to do the construction may give a quicker result than the official solution. It may be, however, not trivial to find this alternative solution.

### 4.2 A Right Triangle (Problem 23)

The problem setting can be seen in Fig. 6
We use some recent features of GeoGebra Discovery to solve this problem. Most importantly, a square is created based on free points $A=(0,0)$ and $B=(1,0)$. They must not be defined with the help of any axes, because in that case the background proof will fail and no output will be obtained. Here, instead of copying the initial square several times, we use the Dilate tool to stretch the segment $A B$ and $A C$ to get $B^{\prime}$ and $C^{\prime}$ accordingly. Another trick is to create the diagonal $j=A D$ of the initial square. Now the intersection $E$ of $k=B^{\prime} C^{\prime}$ and $j$ is the searched point. Finally, projecting $E$ on $n=A B^{\prime}$ and obtaining intersection point $F$ of perpendicular $l$ and line $n$, comparison of $m=E F$ and $f=A B$ is to be done. And, indeed $m=253 / 34$, as expected. (See the sketch in Fig. 7 in GeoGebra Discovery.)

Here we remark that finding the rational value $253 / 34$ (it is approximately 7.44 ) seems very difficult unless one does not solve the problem explicitly (as shown in the official solution, by using an equation). If a user has some routine in GeoGebra, sketching the problem may take a shorter time than finding the required equation (even if it is a linear one). The construction steps can be found in Table 3.

### 4.3 A Triangle and a Circle (Problem 47)

Problem 47 was not even accessible during the contest for most of the teams because it was almost the last problem in the list and they were not that fast.

The problem setting can be seen in Fig. 8 .
The official solution of Problem 47 required some non-trivial ideas. When using GeoGebra Discovery, we may face the question how the problem setting can be constructed, which is shown on Table 4 Since the lengths $A D=8$ and $B D=3$ are given, it seems reasonable to create $A B$ arbitrarily and create $D$ by using the command Dilate (B, 8/11, A). Now, we create another point $A^{\prime}$ in a similar way, by placing $A^{\prime}$ on $A B$ and letting $A A^{\prime}=7$. This helps us restricting the position of $O$ because it must be


Figure 7: Sketch for Problem 23

Problem 47. Let $O$ be the circumcenter of triangle $A B C$. Let further points $D$ and $E$ lie on the segments $A B$ and $A C$, respectively, so that $O$ is the midpoint of $D E$. If $A D=8, B D=3$, and $A O=7$, determine the length of $C E$.
Result. $\frac{4 \sqrt{21}}{7}$
Figure 8: Problem setting 47
on the circumcircle of the circle $c$ with center $A$ and radius $A A^{\prime}$. On the other hand, $O$ must lie on the perpendicular bisector $g$ of $A B$. At this point we already know the position of $O=c \cap g$. (In fact, there may be two solutions here, but they are identical in the sense of symmetry.)

Now, by reflecting $D$ about $O$ we obtain $E$. By having $E$, we already know the line $B C$. To get the point $C$ we only have to intersect this line with the circumcircle $c$. (Again, there are two solutions, but the other one $C^{\prime}$ leads to a degenerate case because it yields $A=C^{\prime}$. To force getting the non-degenerate case we need to click near the intersection point with the mouse. Otherwise GeoGebra Discovery will compute with both cases at the same time.)

As a final step, we designate the unit length. Luckily, $D A^{\prime}$ is exactly 1 . So we just have to compare $j=A E$ and $i=D A^{\prime}$. As expected, the result is $j=4 / 7 \cdot \sqrt{21} \cdot i$. Thus, $C E=j=\frac{4 \sqrt{21}}{7}$.

The sketch can be seen in Fig. 9 .

## 5 Problems that Require Further Improvements

In this section we take an overview of other examples that cannot be fully solved in simple steps in GeoGebra Discovery. Some hints may be, however, obtained. Instead of getting such hints, we summarize how the software tool could be extended to be able to give full solutions for such problems.

A first set of problems ( 4,25 and 58, see Fig. 10,12 and 16) are related with angles, another set (Problems 18, 30, 34, 43 and 58, see Fig. 11, 13, 14, 15, and 16) is about areas. Angle support (via symbolic computation) is very poor in GeoGebra Discovery: this has roots in a non-bijective relationship between angles and their algebraic counterparts. Area support is also somewhat minimal, because it is restricted to triangles, and the expected way of use is not polished yet in the software tool.


Figure 9: Sketch for Problem 47

Problem 4. A square and a regular pentagon are as in the picture below. Find the angle $\alpha$ in degrees.


Figure 10: Problem 4

Problem 18. A rectangle with sides of length 3 and 4 is inscribed into a circle. Moreover, four half-circles are glued to its sides from outside as in the picture. What is the area of the shaded region, which consists of points of the half-circles not lying inside the circle?


Figure 11: Problem 18

Problem 25. Consider a semi-circle with centre $C$ and diameter $A B$. A point $P$ on $A B$ satisfies the following. A laser beam leaves $P$ in a direction perpendicular to $A B$, bounces off the semicircle at points $D$ and $E$ following the rule of reflection, that is, $\angle P D C=\angle E D C$ and $\angle D E C=\angle B E C$, and then it hits the point $B$. Determine $\angle D C P$ in degrees.


Figure 12: Problem 25

Problem 30. Giuseppe bought an ice cream. It had the shape of a ball of radius 4 cm in an ice cream cone. Giuseppe noticed that the ice cream ball fitted in the cone in the following way: The centre of the ball was precisely 2 cm above the base of the cone and the cone surface ended exactly where it touched the ball tangentially. What was the volume of the cone?


Figure 13: Problem 30

Problem 34. Given a triangle $A B C$ of area 1 , extend its sides $B C, C A, A B$ to points $D, E$, and $F$ respectively, as in the figure, so that $B D=2 B C, C E=3 C A$ and $A F=4 A B$. Find the area of the triangle $D E F$.


Figure 14: Problem 34

Problem 43. The medians of triangle $A B C$ dissect it into six sub-triangles. The centroids of these sub-triangles are vertices of hexagon $D E F G H I$. Find the area ratio between the hexagon $D E F G H I$ and the triangle $A B C$.

Figure 15: Problem 43

Problem 58. A point $P$ is located in the interior of triangle $A B C$. If

$$
A P=\sqrt{3}, \quad B P=5, \quad C P=2, \quad A B: A C=2: 1, \quad \text { and } \quad \angle B A C=60^{\circ},
$$

what is the area of triangle $A B C$ ?
Figure 16: Problem 58

One can find that Problems 25 and 58 have some common roots. They can be formulated with "implicit assumptions".

We have a closer look at Problem 25. Let point $F$ be the next bounce, when we assume that a laser beam starts from point $P$. Now, a command like LocusEquation ( $F==B, \angle D C P$ ) could address the question (but having angles in the second parameter is not implemented). In fact, we may already get the exact position of $P$ when applying consecutive reflections. According to Fig. 17, when reflecting $P$ about $C D$, and intersecting the line connecting $D$ and the mirror image $P^{\prime}$ with the semicircle, we can obtain $E$. Another reflection can yield $P^{\prime \prime}$ and the final visualization can be achieved with LocusEquation(AreCollinear ( $E, P^{\prime \prime}, B$ ) , $P$ ). Since GeoGebra Discovery shows 5 isolated points, one can conjecture that there is something to do with a regular pentagon. Here, unfortunately, the factorization of the obtained polynomial does not help, because the interesting quadratic numbers are appearing just approximately. A deeper symbolic study shows that (by assuming $A=(0,0)$ and $B=(1,0)$ ) for the $x$-coordinate of $P$ is one of the roots of the polynomial $64 x^{5}-128 x^{4}+80 x^{3}-17 x^{2}+x$, and they are

$$
0, \frac{1}{16} \cdot(-2 \cdot \sqrt{5}+6), \frac{1}{4}, \frac{1}{16} \cdot(2 \cdot \sqrt{5}+6), 1,
$$

and to these values belong the $\alpha$ values

$$
0^{\circ}, 36^{\circ}, 60^{\circ}, 288^{\circ}, 360^{\circ},
$$

the latter two ones without real geometrical meaning. Finally we can conclude that $\alpha=36^{\circ}$, this is the only meaningful solution. But, all of this derivation requires some additional steps, GeoGebra Discovery alone does not bring a satisfactory final answer.

Finally, we show a wrong conjecture for Problem 58, based on GeoGebra Discovery. Like Problem 25 , an implicit locus equation seems here helpful. Let us, first, create a regular triangle $A B^{\prime} C$ with $A=(0,0), B=(1,0)$, and reflect $A$ about $B^{\prime}$ to get $B$. Clearly, these preparations are sufficient to ensure assumptions $A B: A C=2: 1$ and $\angle B A C=60^{\circ}$ (See Fig. 18). This dummy triangle has the area $\frac{\sqrt{3}}{2}$. Now, we create an arbitrary point $P$ and connect it with points $A, B$ and $C$, to get segments $i, j$ and $k$, respectively. We create two locus equations with the commands LocusEquation ( $j / k==5 / 2, P$ ) and LocusEquation $(i / k==$ sqrt (3) $/ 2, P$ ). Now, we want to find the correct position for $P$, so we consider the intersection of the two locus curves visually. After zooming in, we learn that for $P=$ ( $0.4739140532,0.24828147621$ ) we obtain $k=0.618294458$ which seems to be close enough to the well known number $f=\frac{\sqrt{5}-1}{2}$. If so, the triangle must be enlarged by a factor $1 / f \cdot 2$ which is twice the golden ratio, $2 \varphi=\sqrt{5}+1$. Finally, the triangle will have the area $\frac{\sqrt{3}}{2} \cdot(\sqrt{5}+1) \approx 9.06913 \ldots$

Assuming that the golden ratio plays a role here is, however, incorrect. The correct solution is $\frac{6+7 \cdot \sqrt{3}}{2} \approx 9.06217 \ldots 5$ Note that the ratio between the two values is $1.000767 \ldots$ which is remarkably small difference.

[^2]

Figure 17: A possible approach to solve Problem 25


Figure 18: Obtaining an incorrect conjecture to solve Problem 58

## 6 Conclusion

The last section showed that GeoGebra Discovery can be a useful tool to get a correct conjecture if the right steps are taken to finish the solution, but it can also be misleading in some delicate situations. On the other hand, several contest problems can be handled and solved with minimal effort by using this tool. We need to admit that a good knowledge of the software is unavoidable. However, experienced users may need just a couple of steps to achieve the solution.

For a future improvement, full support of computing angles and areas seems to be a great step forward. Some sophisticated problems, however, may need further developments towards symbolic computations that are based on implicit assumptions.

## 7 Acknowledgements

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## References

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## 8 Appendix

In this Appendix, we provide the construction protocols in GeoGebra Discovery for the problems presented above.

Table 1: Construction protocol in GeoGebra Discovery for Problem 6

| No. | Name | Toolbar Icon | Description |
| :---: | :---: | :---: | :---: |
| 1 | Polygon poly1 | $\because$ | Polygon(A, B, 4) |
| 2 | Point E | - | Midpoint of A, C |
| 3 | Point E' | ${ }^{\circ}$ | E mirrored at A |
| 4 | Point E'_1 | $\bullet^{\circ}$ | E mirrored at B |
| 5 | Polygon poly2 | $\square$ | Polygon(E', E'_1, 4) |
| 6 | Point E'_2 | . ${ }^{\circ}$ | E mirrored at $\mathrm{E}^{\prime}$ |
| 7 | Point E'_3 | .$^{\circ}$ | E mirrored at E'_1 |
| 8 | Polygon poly3 | $\square$ | Polygon(E'_2, E'_3, 4) |
| 9 | Segment t | $\bigcirc$ | Segment E'_2, E'_3 |

Table 1: Construction protocol in GeoGebra Discovery for Problem 6

| No. | Name | Toolbar Icon | Description |
| ---: | :--- | :---: | :--- |
| 10 | Segment s | $\ddots$ | Segment A, C |
| 11 | Number P |  | 4 t |

Table 2: Construction protocol for Problem 15

| No. | Name | Toolbar Icon | Description | Value |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Polygon poly1 | $\because$ | Polygon(A, B, 4) | poly $1=7.33$ |
| 2 | Segment f | $\bigcirc$ | Segment A, B | $f=2.71$ |
| 3 | Polygon poly2 | $\because$ | Polygon(D, C, 4) | poly $2=7.33$ |
| 4 | Polygon poly 3 | $\square$ | Polygon(C, B, 4) | poly $3=7.33$ |
| 5 | Polygon poly4 | $\because$ | Polygon(A, D, 4) | poly $4=7.33$ |
| 6 | Polygon poly5 | $\because$ | Polygon(B, A, 4) | poly $5=7.33$ |
| 7 | Segment g_1 | $\bigcirc$ | Segment E, K | $g_{1}=8.56$ |
| 8 | Polygon poly6 | $\because$ | Polygon(I, D, 4) | poly $6=7.33$ |
| 9 | Segment 1_1 | $\bigcirc$ | Segment N, H | $l_{1}=8.56$ |
| 10 | Point O | 大 | Intersection of g_1 and 1_1 | $O=(1.18,3.29)$ |
| 11 | Segment m_1 | $\bigcirc$ | Segment O, E | $m_{1}=1.71$ |

Table 3: Construction protocol for Problem 23

| No. | Name | T. Icon | Description | Value |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Point A | - ${ }^{\text {a }}$ |  | $A=(0,0)$ |
| 2 | Point B | - ${ }^{\text {a }}$ |  | $B=(1,0)$ |
| 3 | Point B' | 0 | B dilated by factor 23 from A | $B^{\prime}=(23,0)$ |
| 4 | Segment n | $\bigcirc$ | Segment A, B' | $n=23$ |
| 5 | Polygon poly1 | 9 | Polygon(B, A, 4) | poly $1=1$ |
| 6 | Segment f | $\bigcirc$ | Segment B, A | $f=1$ |
| 7 | Line j | $\bigcirc$ | Line D, A | $j:-x-1 y=0$ |
| 8 | Point C' | .0 | C dilated by factor 11 from A | $C^{\prime}=(0,-11)$ |
| 9 | Segment k | $\bigcirc$ | Segment C', B' | $k=25.5$ |
| 10 | Point E | > | Intersection of j and k | $\begin{aligned} & E \\ & (7.44,-7.44) \end{aligned}=$ |
| 11 | Line 1 | + | Line through E perpendicular to n | $l: x=7.44$ |

Table 3: Construction protocol for Problem 23

| No. | Name | T. Icon | Description | Value |
| ---: | :--- | :---: | :--- | :--- |
| 12 | Point F | $>$ | Intersection of l and n | $F=(7.44,0)$ |
| 13 | Segment m | $\sigma$ | Segment F, E | $m=7.44$ |

Table 4: Construction protocol for Problem 47

| No. | Name | T. Icon | Description | Value |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Point B | - ${ }^{\text {a }}$ |  | $B=(-5.41,-5.8)$ |
| 2 | Point A | - ${ }^{\text {a }}$ |  | $A=(-0.78,4.18)$ |
| 3 | Line g | $\cdots$ | Perpendicular Bisector of AB | $\begin{aligned} & g: 4.63 x+9.98 y= \\ & -22.4 \end{aligned}$ |
| 4 | Point D | 0 | B dilated by factor $8 / 11$ from A | $\begin{aligned} & D \\ & (-4.15,-3.08) \end{aligned}=$ |
| 5 | Point A' | ${ }^{\circ}$ | A dilated by factor $1 / 8$ from D | $\begin{aligned} & A^{\prime} \\ & (-3.73,-2.17) \end{aligned}=$ |
| 6 | Circle c | $\odot$ | Circle through A' with center A | $\begin{aligned} & c:(x+0.78)^{2}+ \\ & (y-4.18)^{2}=49.04 \end{aligned}$ |
| 7 | Point O | - | Intersection of cand g | $O=(0.84,-2.63)$ |
| 8 | Circle d | $\odot$ | Circle through A with center O | $\begin{aligned} & d:(x-0.84)^{2}+ \\ & (y+2.63)^{2}=49.04 \end{aligned}$ |
| 9 | Point E | .${ }^{\circ}$ | D mirrored at O | $E=(5.82,-2.19)$ |
| 10 | Line h | 8 | Line A, E | $\begin{aligned} & h: 6.37 x+6.6 y= \\ & 22.63 \end{aligned}$ |
| 11 | Segment f | $\cdots$ | Segment A, B | $f=11$ |
| 12 | Segment i | $\bigcirc$ | Segment D, A' | $i=1$ |
| 13 | Point C | $x$ | Intersection of d and h | $C=(7.7,-4.01)$ |
| 14 | Segment j | $\cdots$ | Segment C, E | $j=2.62$ |


[^0]:    ${ }^{1}$ See https://telex.hu/tech/2023/05/09/chatgpt-bing-mesterseges-intelligencia-matematika-erett segi
    ${ }^{2}$ See Bill Gates' notes at https://www.gatesnotes.com/ASU-and-GSV?WT.mc_id=20230419100000_ASU-GSV-202 3_BG-EM_
    ${ }^{3}$ See Stephen Wolfram's notes at https://writings.stephenwolfram.com/2023/03/chatgpt-gets-its-wolfram -superpowers/.

[^1]:    ${ }^{4}$ GeoGebra Discovery is freely available at https://kovzol.github.io/geogebra-discovery

[^2]:    ${ }^{5}$ See https://math.old.naboj.org/archive/problems/pdf/math/2023_en_sol.pdf for a full computation.

